Mind Your Path: On (Key) Dependencies in Differential Characteristics

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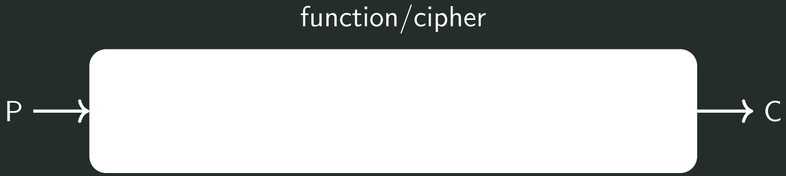
FSE 2023

Outline

Differential cryptanalysis



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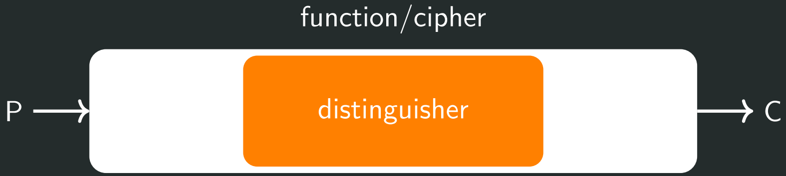

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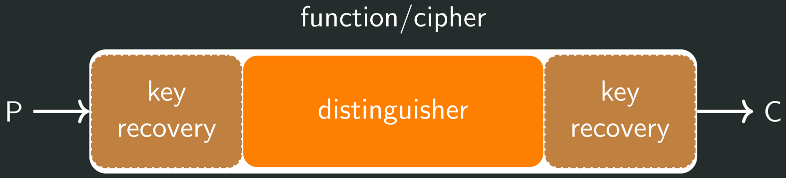

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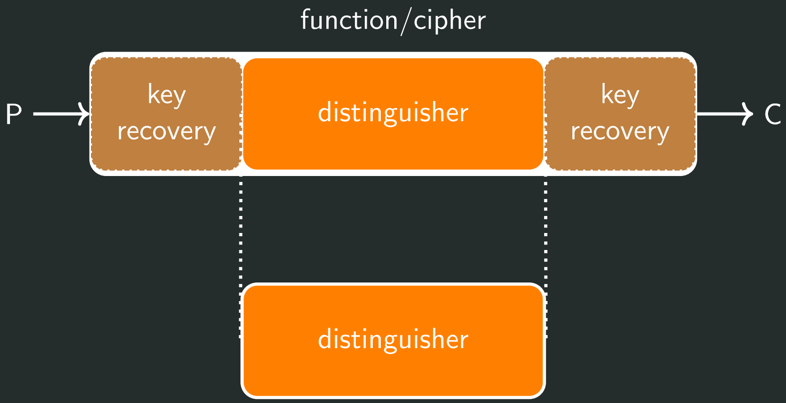

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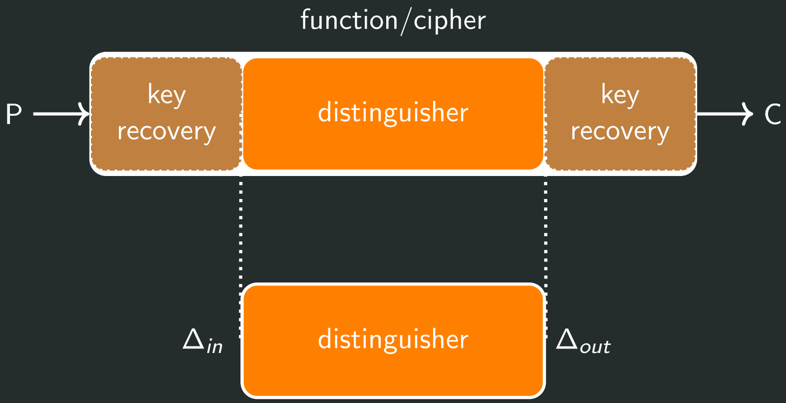

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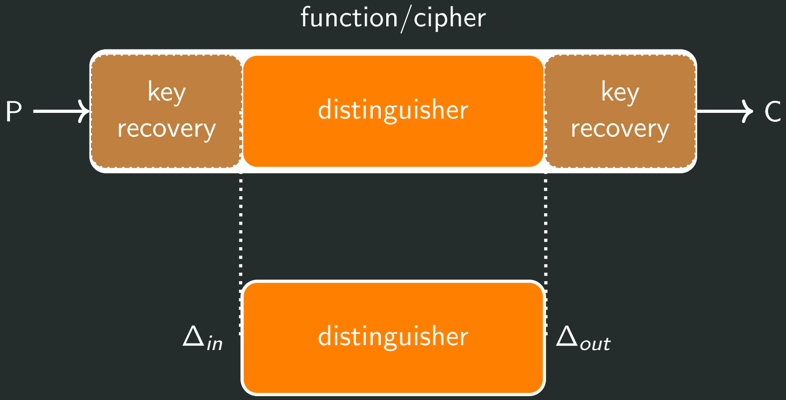

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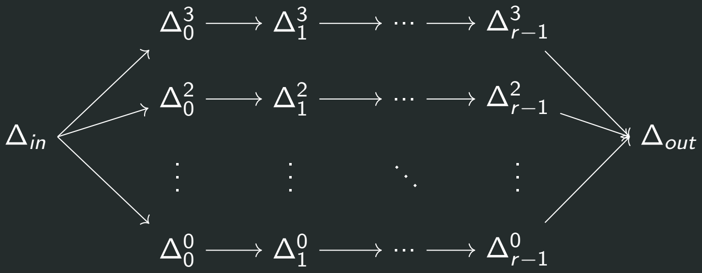

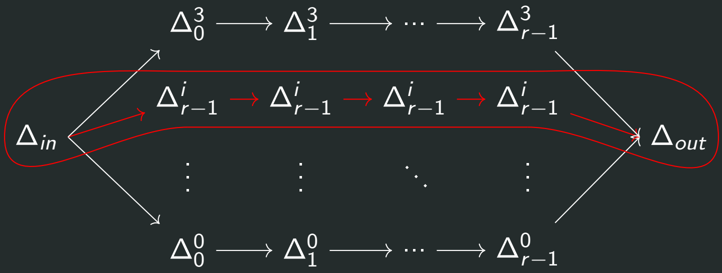

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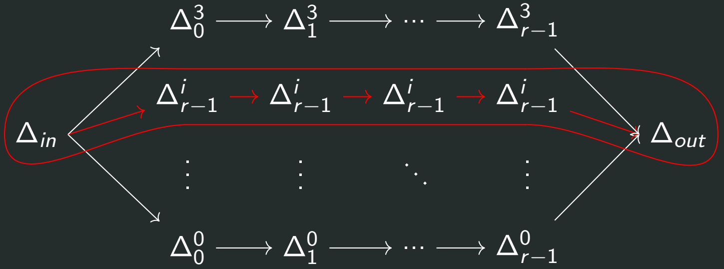

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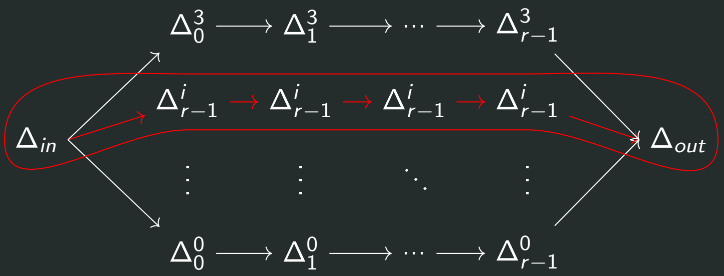
$$\begin{aligned}\mathbb{P}(\Delta_{in} \to \Delta_{out}) &= \sum_i \mathbb{P}(\Delta_{in} \to \Delta_1^i \to \ldots \to \Delta_{r-2}^i \to \Delta_{out}) \\ &\geq \mathbb{P}(\Delta_{in} \to \Delta_1^* \to \ldots \to \Delta_{r-2}^* \to \Delta_{out})\end{aligned}$$



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Differential probability of a round function is independent of the value, assuming the subkey k is uniformly random [LMM91]. Under this assumption,

$$\mathbb{P}(\Delta_{in} \rightarrow \Delta_1^* \rightarrow ... \rightarrow \Delta_{r-2}^* \rightarrow \Delta_{out})$$
$$\approx \mathbb{P}(\Delta_{in} \rightarrow \Delta_1^*) * \mathbb{P}(\Delta_1^* \rightarrow \Delta_2^*) * ... * \mathbb{P}(\Delta_{n-2}^* \rightarrow \Delta_{out})$$

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- Difference Distribution Table
- Automated methods (SAT,MILP,CP)

Related works

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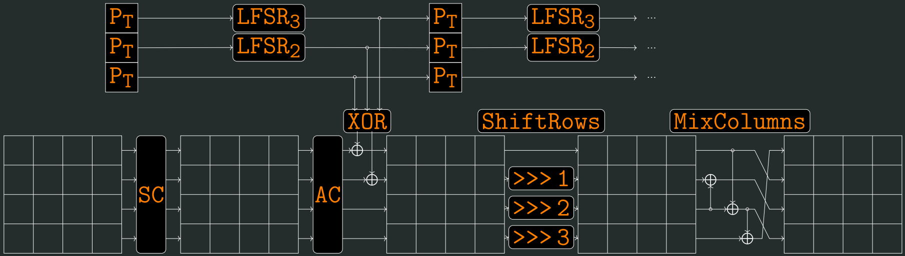
- On ARX/RX ciphers [SRB21, Leu12, XLJ+22]

SKINNY round function [BJK+16]



- Block size $n = 64$ or 128 bits
- Tweakable block cipher (tweakey size is $n, 2n$ or $3n$)

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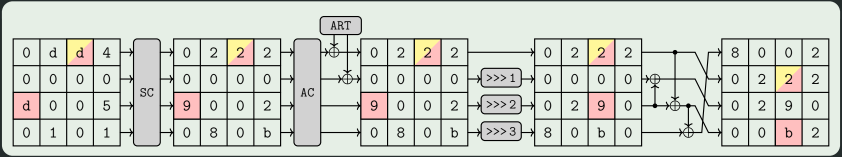
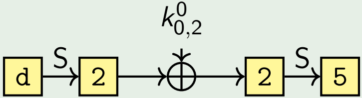
- For dependencies that are not too complex, we want to approximate the size of the valid key space
- A search method for differential characteristics that also avoid some of these key dependencies (particularly those that invalidate them)

Linear constraints





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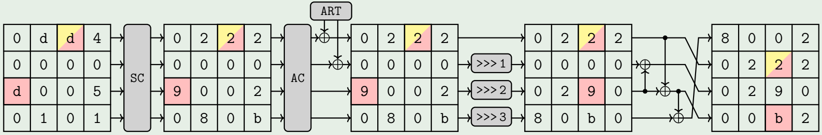

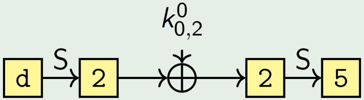

$$\mathcal{Y}_{DDT}(\texttt{0xd}, \texttt{0x2}) = \{\texttt{4,6,c,e}\}$$

$$\mathcal{X}_{DDT}(\texttt{0x2}, \texttt{0x5}) = \{\texttt{0,2,9,b}\}$$

$$\implies k^0_{0,2} \in \{\texttt{4,5,6,7,c,d,e,f}\}$$

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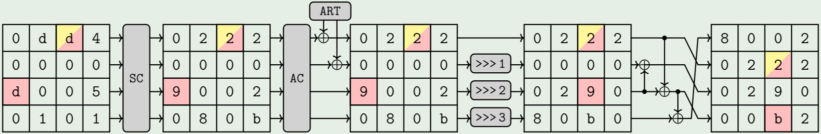

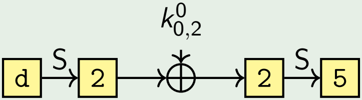

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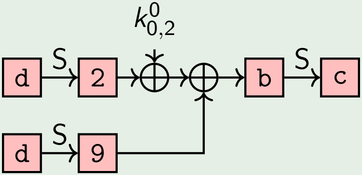
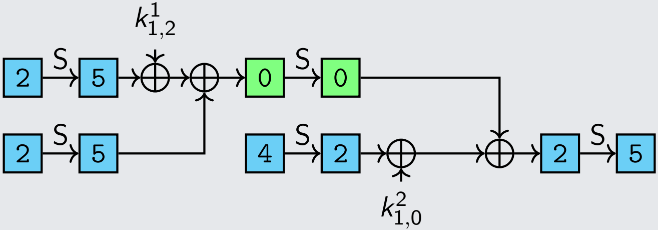


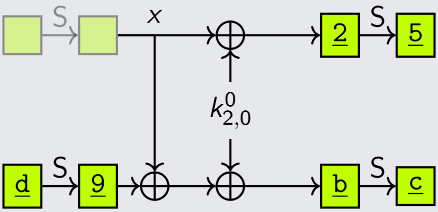
$$k^0_{0,2} \in \{\texttt{0,1,2,3,8,9,a,b}\}$$

Nonlinear constraints



$(k^1_{1,2}, k^2_{1,0}) \in \ldots$

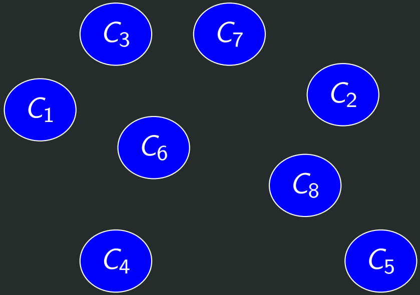
Higher-order constraints



$$x \oplus k^0_{2,0} \in \mathcal{X}_{DDT}(\texttt{0x2}, \texttt{0x5})$$

$$x \oplus k^0_{2,0} \oplus y \in \mathcal{X}_{DDT}(\texttt{0x2}, \texttt{0x5}) \text{ where } y \in \mathcal{Y}_{DDT}(\texttt{0xd}, \texttt{0x9})$$

Combining constraints



these constraints (may) limit the possible key space and change the probability distribution

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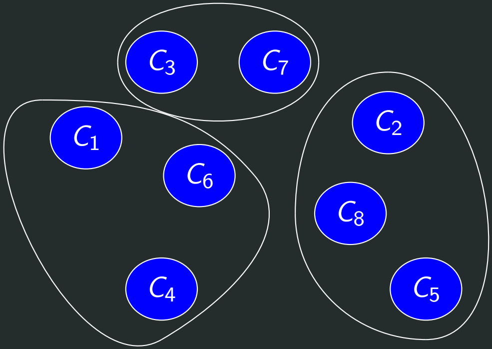

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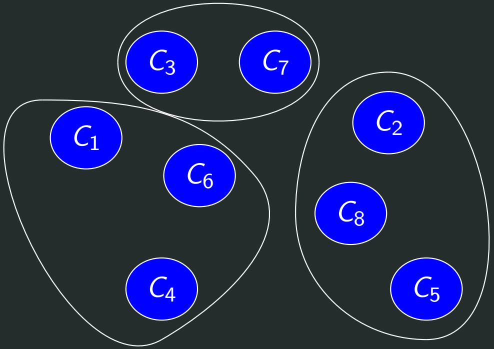

these constraints (may) limit the possible key space and change the probability distribution

C_i and C_j are in the same group if at least one of the following conditions is fulfilled:

- They share at least one key cell (up to key schedule)
- They share at least one Sbox

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- This ensures that within the first 30 rounds, after applying a constraint on the XORed key,
  - All XORed keys are still possible after an application of LFSR
  - The key distribution is uniform

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When the groups are small, we can compute the change in probability distribution

- If we are dealing with TK2/TK3, we can split a group further
  - $k_i^n = tk_{i,1}^n \oplus tk_{i,2}^n$
- If only one Sbox is common, we can use a hash-table to record the values/distribution that C_i allows, then use it to compute C_j

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Otherwise, we can conduct an experimental search

A summary of results for SKINNY

- SKINNY-64

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  - We can plot the estimated theoretical probability distribution

Prob. in $-\log_2$	35.415	36.415	37	37.415	38	39
Percentage	5.56%	22.2%	5.56%	22.2%	22.2%	22.2%

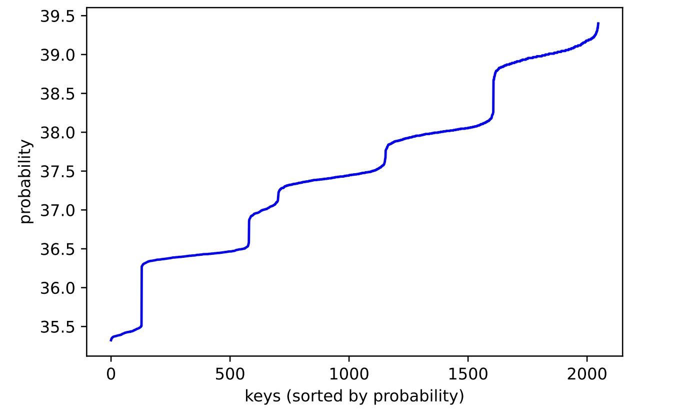

Figure 1: Experimental probability distribution across 2048 random but valid keys

A summary of results for SKINNY

- SKINNY-64
  - 10 out of 21 tested differential characteristics are impossible
  - The remaining differential characteristics work for a small key space
  - We can plot the estimated theoretical probability distribution
- SKINNY-128
  - 11 out of 22 differential characteristics are impossible

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  - Experimentally determined probability distribution

GIFT [BPP+17]

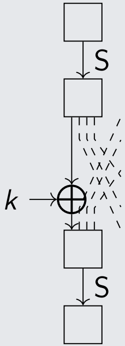

Figure 2: Linear constraint

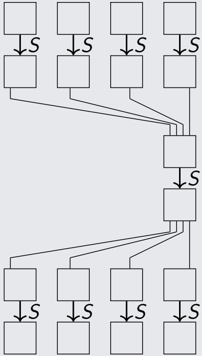

Figure 3: Nonlinear constraints

A summary of results for GIFT

For GIFT-64 and GIFT-128,

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For GIFT-64 and GIFT-128,

- 1 out of 15 tested differential characteristics is impossible

A summary of results for GIFT

For GIFT-64 and GIFT-128,

- 1 out of 15 tested differential characteristics is impossible
- Most of the remaining tested differential characteristics have key-dependent constraints

Impact on differentials

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However

- Probability of the dominant characteristic may change
- Experiments show that there is a possibility that there is no valid keys for all the differential characteristics in a differential

Integrating with Constraint Programming (CP)

- Looking for right pairs directly might be hard in some scenarios

Integrating with Constraint Programming (CP)

- Looking for right pairs directly might be hard in some scenarios
- Incorporate additional constraints in CP which uses the input and output values of active Sboxes to verify the validity of the propagation.

Thank you for you attention!

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