Radboud University

EliMAC: Speeding Up LightMAC by Around 20%

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FSE 2024

March 25, 2024

ESCADA

Introduction

- Using key K, message M is signed with tag T
- Verification takes K and (M, T) and outputs $\begin{cases} \top \text{ if tag is correct} \\ \bot \text{ if tag is incorrect} \end{cases}$

Message Authentication Codes

$$M \xrightarrow[*]{} \boxed{\mathrm{MAC}_K} \xrightarrow[t]{} T$$

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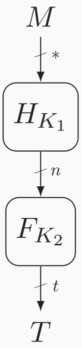
- Verification takes K and (M, T) and outputs $\begin{cases} \top \text{ if tag is correct} \\ \bot \text{ if tag is incorrect} \end{cases}$
- Security goal: unforgeability
- Often, one adopts a stronger notion: PRF security
  - MAC_K should behave like a random function
  - $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{MAC}}(q)$ should be small

Hash-then-PRF

Idea

- Process arbitrary length M through "weaker" universal hash H_{K_1}
- Protect with "stronger" F_{K_2}



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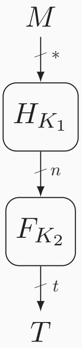

Security

- Secure MAC function if
  - H is ϵ-universal hash function
  - F is pseudorandom function
- F can be replaced by (truncation of) block cipher E at some loss
- Extra message block can be entered after H if it is ϵ-XOR-universal

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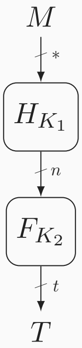

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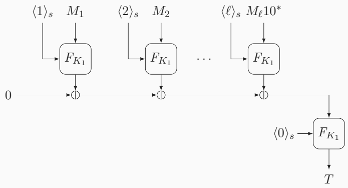
Ideally: H_{K_1} is parallelizable

Protected counter sum [Ber99]



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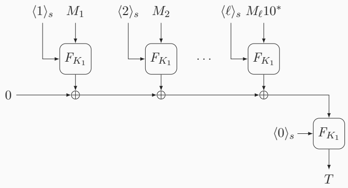
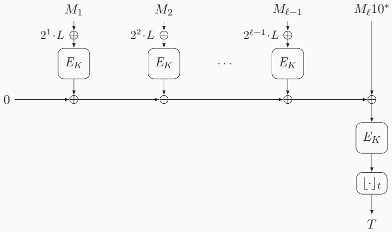


PMAC [BR02] $(L = E_K(0^n))$



Examples of Parallelizable Hash-then-PRF Designs

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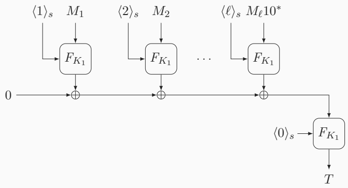

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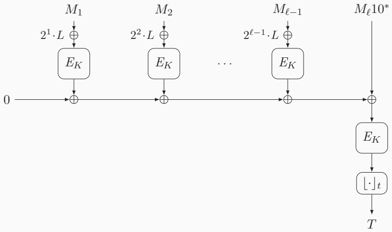
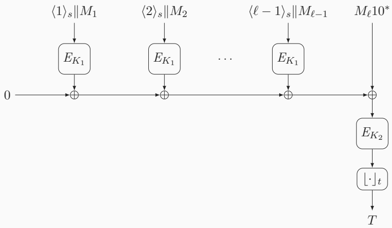


LightMAC [LPTY16]



“Expensive” Parallelizable Universal Hashing

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Goal: parallelizable domain extender for universal hashing

1. EliHash: fully parallelizable universal hash from fixed-length hashes
2. EliMAC: MAC function on top of EliHash
3. Instantiation of EliMAC using round-reduced AES
4. Side-result: flaws in earlier attempt Marvin [SBB$^+$09, SB12]

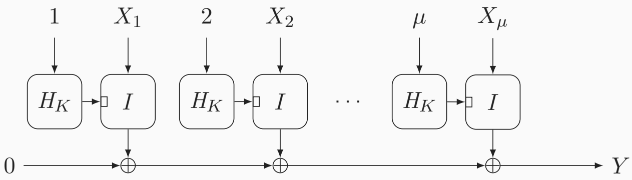
EliHash and EliMAC

Building Blocks

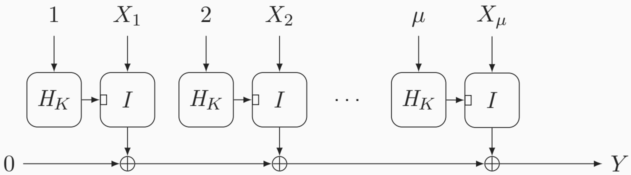
- Two – not necessarily independent – families of hash functions:
  - $H : \mathcal{K}' \times [1, \ldots, \mu] \to \mathcal{K}$
  - $I : \mathcal{K} \times \mathcal{X} \to \mathcal{Y}$

Design

- EliHash : $\mathcal{K}' \times \mathcal{X}^{[1\ldots\mu]} \to \mathcal{Y}$ is defined as



EliHash: Security (1/2)



ε-XOR-universality

- For any distinct X, X' and any Y: $\mathbf{Pr}_K\left(\mathcal{H}_K(X) \oplus \mathcal{H}_K(X') = Y\right) \leq \varepsilon$

Goal

- XOR-universality of EliHash as long as H and I satisfy certain conditions

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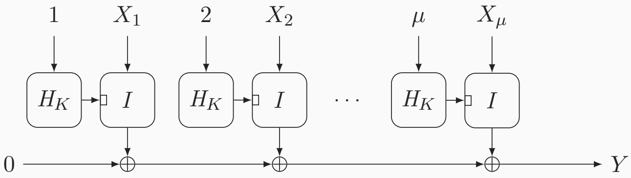

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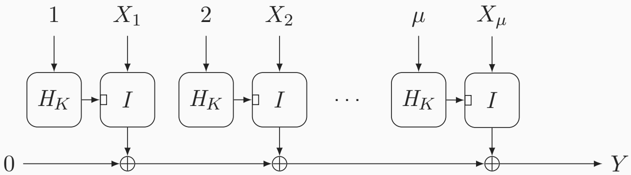

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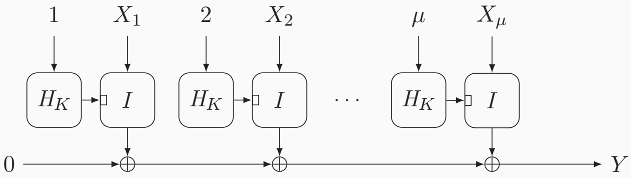

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- Typically more than two evaluations of I are XORed
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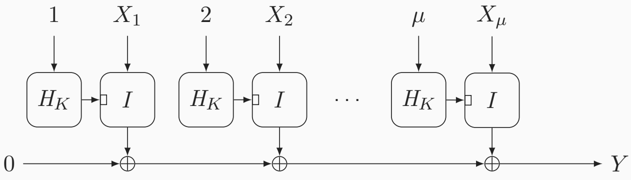
- Okay in our case as H has very small domain: $[1, \ldots, \mu]$



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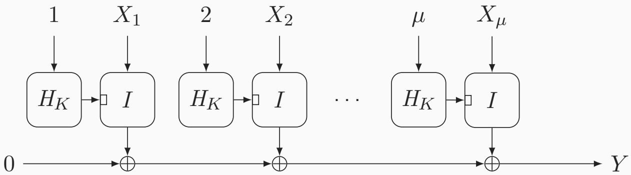
General Result

- Let $\mu \in \mathbb{N}$ be maximal message length to EliHash
- Let $H : \mathcal{K}' \times [1, \ldots, \mu] \to \mathcal{K}$ be δ-μ-independent
- Let $I : \mathcal{K} \times \mathcal{X} \to \mathcal{Y}$ be ε-XOR-universal

EliHash: Security (2/3)



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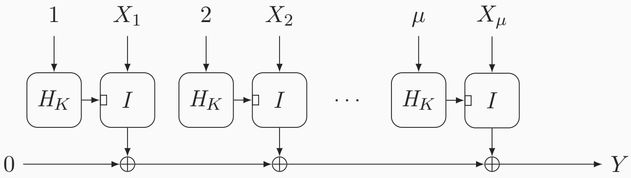
- Then, EliHash : $\mathcal{K}' \times \mathcal{X}^{[1 \ldots \mu]} \to \mathcal{Y}$ is $(|\mathcal{K}|\delta)^\mu \varepsilon$-XOR-universal

EliHash: Security (3/3)



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Rough Proof Idea

- Consider any distinct $\boldsymbol{X} = (X_1, \ldots, X_\ell)$, $\boldsymbol{X}' = (X'_1, \ldots, X'_{\ell'})$ and any Y
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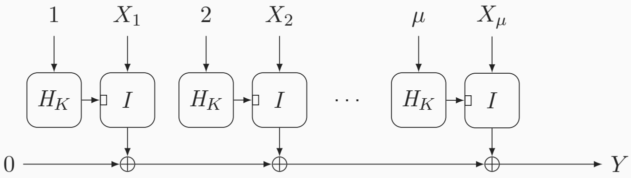

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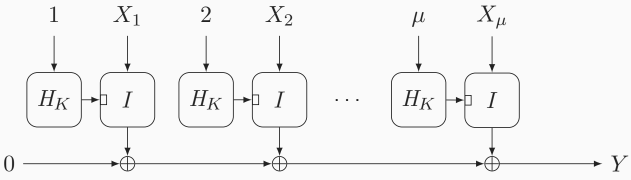

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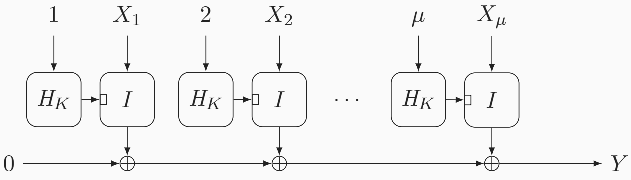

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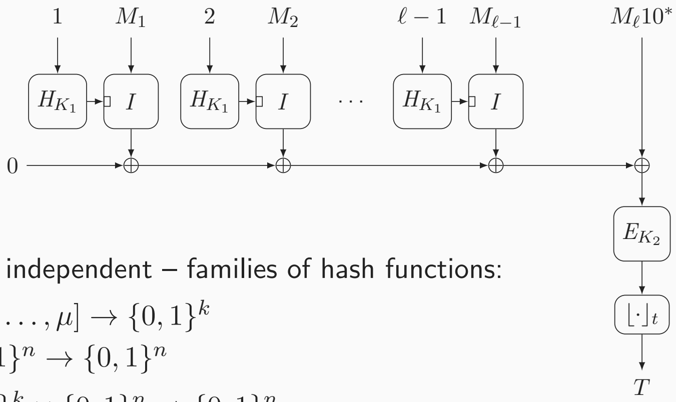
  3. Bound the number of possible key tuples: $\leq (|\mathcal{K}|)^\mu \varepsilon$



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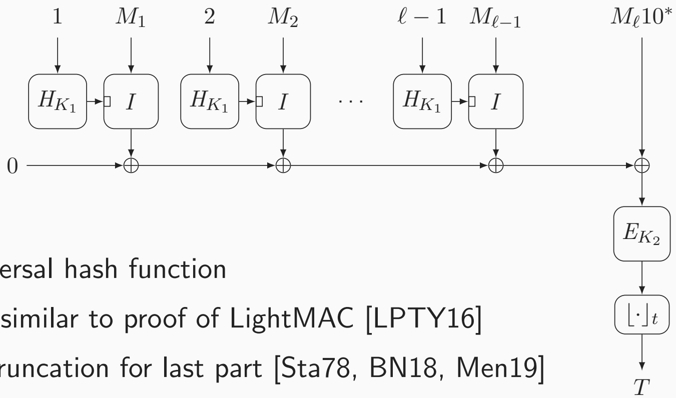
- Block cipher $E : \{0,1\}^k \times \{0,1\}^n \to \{0,1\}^n$

Design

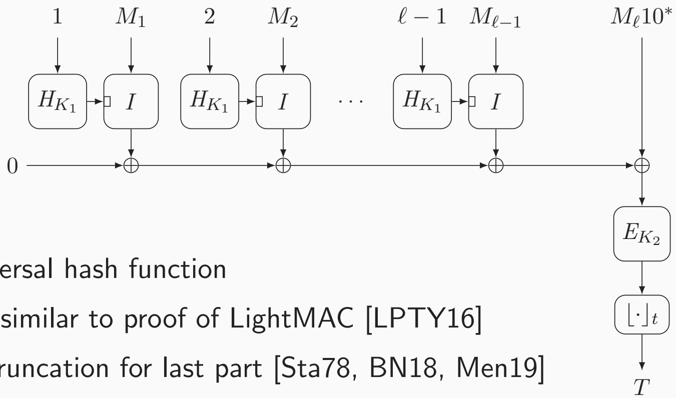
- EliMAC $\approx$ LightMAC but with hashing part replaced by EliHash

EliMAC: Security



Security Proof

- Views EliHash as universal hash function
- Composition to MAC similar to proof of LightMAC [LPTY16]
- Relies on security of truncation for last part [Sta78, BN18, Men19]



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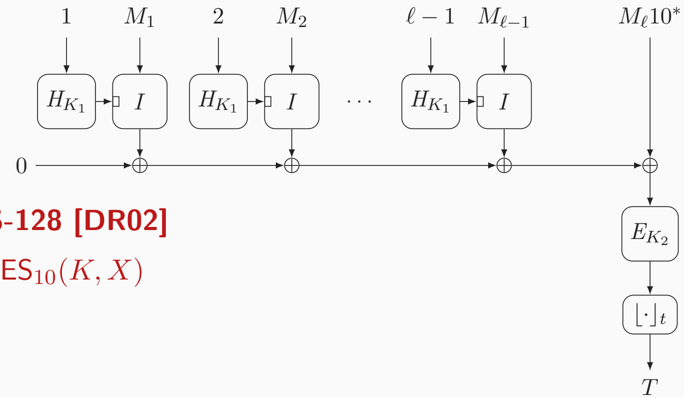
Tightness

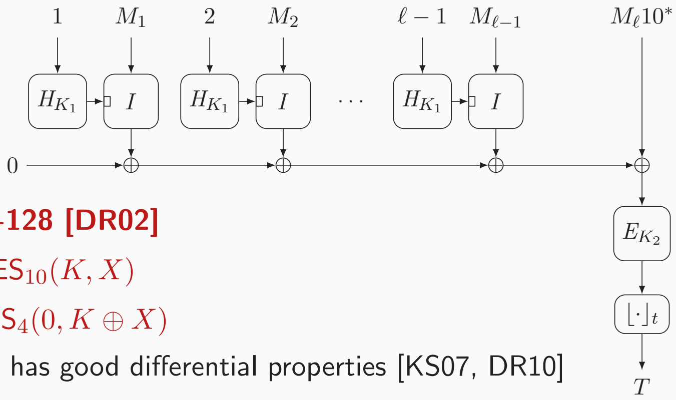
- Matching attacks given in paper

Instantiation



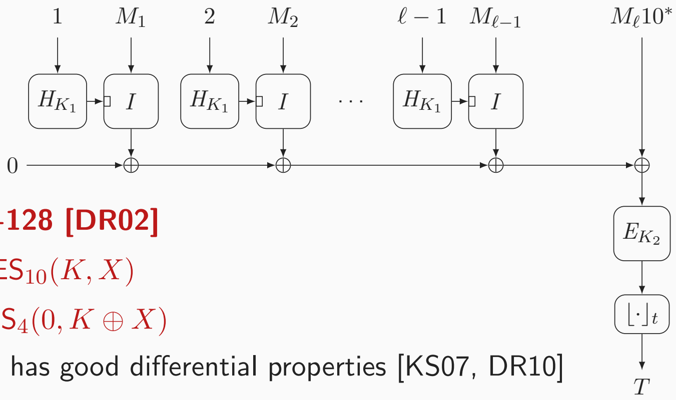
Instantiation Using AES-128 [DR02]

- Instantiation of E: $\mathsf{AES}_{10}(K, X)$



Instantiation Using AES-128 [DR02]

- Instantiation of E: AES$_{10}(K, X)$
- Instantiation of I: AES$_4(0, K \oplus X)$
  - 4-round AES-128 has good differential properties [KS07, DR10]



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- Instantiation of H: $\text{AES}_7(K, \langle i\rangle_{32} \| \langle i\rangle_{32} \| \langle i\rangle_{32} \| \langle i\rangle_{32})$
  - μ-independence does not follow from XOR-universality
  - It appears that 7 rounds suffice [DFJ13] for $\mu \leq 2^{32}$

Theoretical Comparison

scheme	# AES rounds for ℓ blocks		
	pre	online	total
LightMAC	0	10ℓ	10ℓ
EliMAC	$7(\ell-1)$	$4(\ell-1)+10$	$11\ell-1$

- Here, $\ell \leq \mu = 2^{32}$, and counter values encoded using $s = 32$ bits
- EliMAC invokes slightly more AES rounds than LightMAC

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- Note: difference in
  - assumptions (on round-reduced AES for EliMAC) and
  - generic security bounds (64-bit versus 56-bit)

Comparison

- EliMAC-AES: default and with key precomputation
- Comparison with:
  - LightMAC-AES [LPTY16]
  - PMAC2-AES [CCJN21]
  - ZMAC-Deoxys-TBC-256 [IMPS17, JNPS21]
- All parallelizable and with length independent bounds
- Cpb when authenticating $64/1536/4096$ byte messages

		64	1536	4096
Ivy Bridge	LightMAC	3.43	1.13	1.11
	EliMAC	2.18	1.02	0.98
	EliMAC p.c.	2.00	0.46	0.43
	PMAC2	4.50	1.28	1.22
	ZMAC	5.70	1.49	1.26
Broadwell	LightMAC	8.75	0.98	1.08
	EliMAC	1.94	0.76	0.74
	EliMAC p.c.	1.75	0.30	0.27
	PMAC2	3.25	1.13	1.09
	ZMAC	6.97	1.34	1.23
Skylake	LightMAC	2.53	0.86	0.85
	EliMAC	1.56	0.70	0.69
	EliMAC p.c.	1.31	0.27	0.26
	PMAC2	1.71	0.67	0.64
	ZMAC	4.64	0.91	0.84
Zen 2	LightMAC	2.18	0.58	0.58
	EliMAC	1.31	0.45	0.42
	EliMAC p.c.	0.87	0.14	0.13
	PMAC2	1.31	0.58	0.56
	ZMAC	4.34	0.88	0.81

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Notes

- Security assumptions and bounds differ
- Key precomputation ("EliMAC p.c.") is much faster but comes with added memory requirements

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Broadwell	LightMAC	8.75	0.98	1.08
	EliMAC	1.94	0.76	0.74
	EliMAC p.c.	1.75	0.30	0.27
	PMAC2	3.25	1.13	1.09
	ZMAC	6.97	1.34	1.23
Skylake	LightMAC	2.53	0.86	0.85
	EliMAC	1.56	0.70	0.69
	EliMAC p.c.	1.31	0.27	0.26
	PMAC2	1.71	0.67	0.64
	ZMAC	4.64	0.91	0.84
Zen 2	LightMAC	2.18	0.58	0.58
	EliMAC	1.31	0.45	0.42
	EliMAC p.c.	0.87	0.14	0.13
	PMAC2	1.31	0.58	0.56
	ZMAC	4.34	0.88	0.81

Conclusion

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EliHash and EliMAC

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Thank you for your attention!

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