Radboud University

Understanding the Duplex and Its Security

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Radboud University (The Netherlands)
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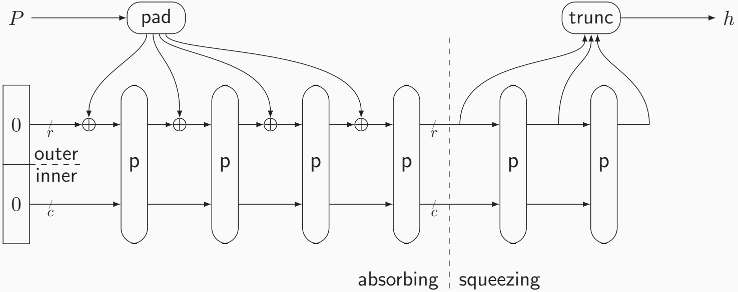
ESCADA

History on Sponges and Duplexes

Sponges [BDPV07]



- p is a b-bit permutation, with $b = r + c$
  - r is the rate
  - c is the capacity (security parameter)
- SHA-3, XOFs, lightweight hashing, ...
- Behaves as RO up to query complexity $\approx 2^{c/2}$ [BDPV08]

Keying Sponges

Keyed Sponge

- $\text{PRF}(K, P) = \text{sponge}(K \| P)$
- Message authentication with tag size t: $\text{MAC}(K, P, t) = \text{sponge}(K \| P, t)$
- Keystream generation of length ℓ: $\text{SC}(K, D, \ell) = \text{sponge}(K \| D, \ell)$
- (All assuming K is fixed-length)

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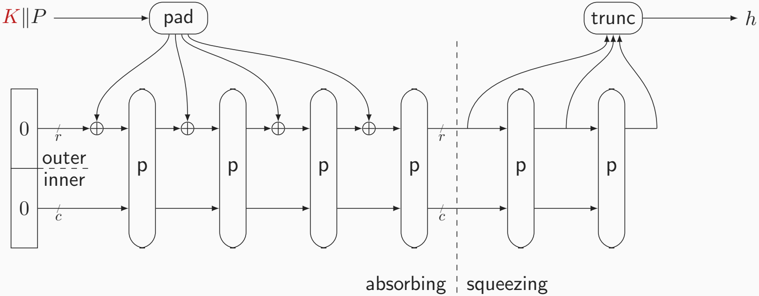
Keyed Duplex

- Authenticated encryption
- Multiple CAESAR and NIST LWC submissions

Evolution of Keyed Sponges



- Outer-Keyed Sponge [BDPV11b, ADMV15, NY16, Men18]

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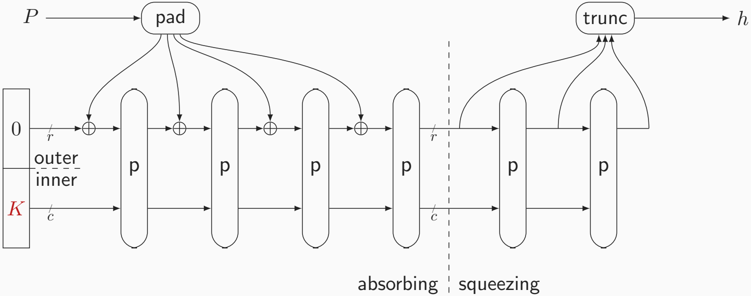

- Outer-Keyed Sponge [BDPV11b, ADMV15, NY16, Men18]
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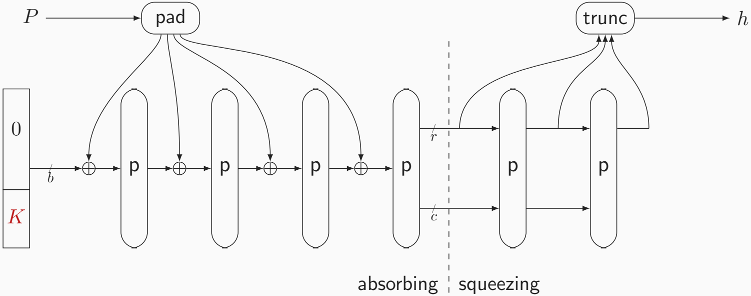
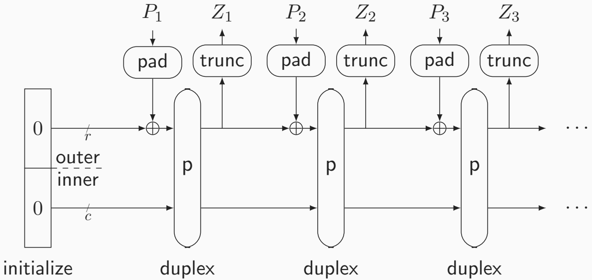


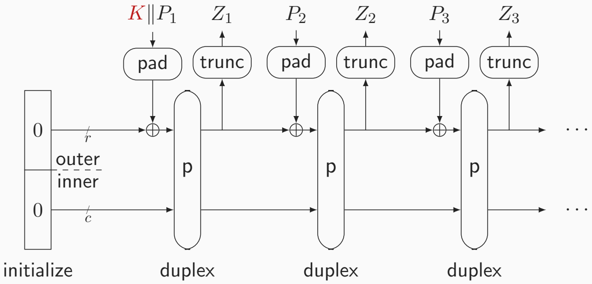
- Outer-Keyed Sponge [BDPV11b, ADMV15, NY16, Men18]
- Inner-Keyed Sponge [CDH$^+$12, ADMV15, NY16]
- Full-Keyed Sponge [BDPV12, GPT15, MRV15]

Evolution of Keyed Duplexes



- Unkeyed Duplex [BDPV11a]

Evolution of Keyed Duplexes



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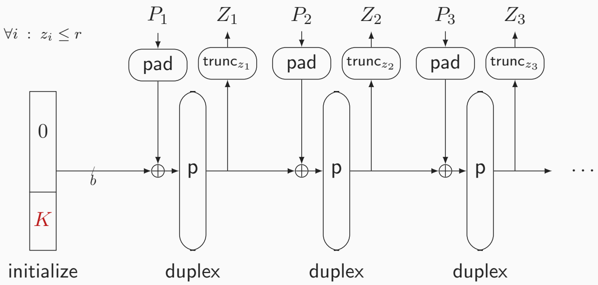

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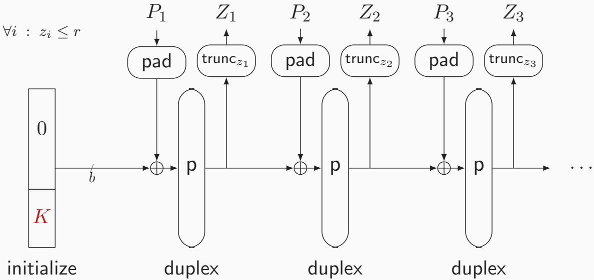

- Unkeyed Duplex [BDPV11a]
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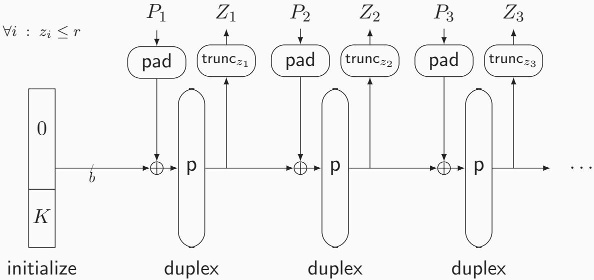
Full-Keyed Duplex of [MRV15] (1)



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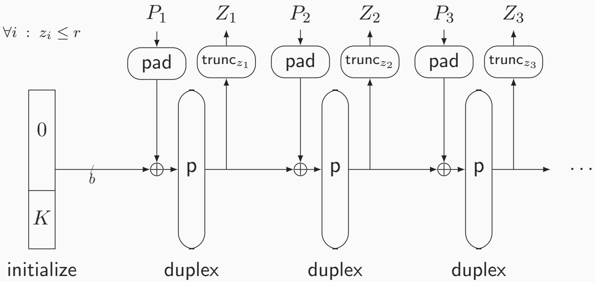

- M: data complexity (calls to construction)
- N: time complexity (calls to primitive)
- $\mu \leq 2M$: multiplicity ("maximum outer collision of p")

Simplified Security Bound

$$\frac{\mu N}{2^k} + \frac{M^2}{2^c}$$

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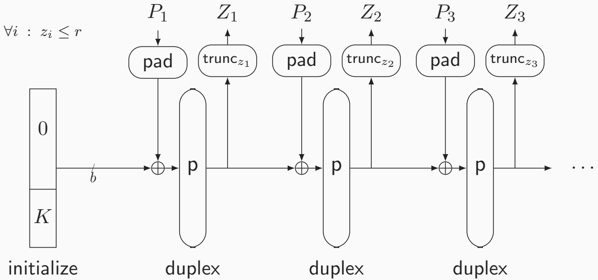

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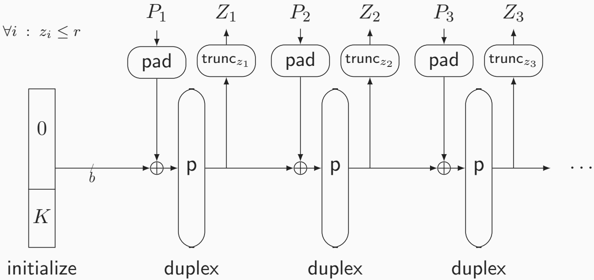
scheme behaves "randomly" as long as this term is $\ll 1$

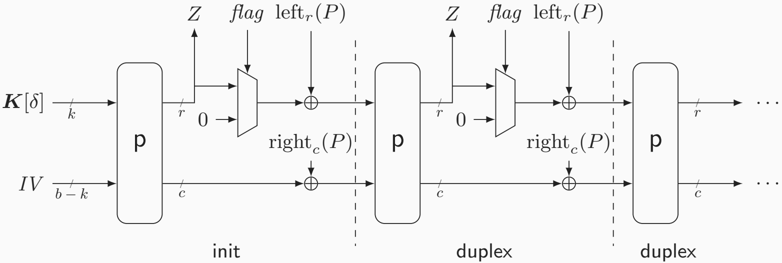
Full-Keyed Duplex of [MRV15] (2)



Limitations

- Multiplicity μ only known a posteriori
- Dominating term $\mu N/2^k$ rather than $\mu N/2^c$
- Limited flexibility in modeling adversarial power
  (multi-user security, blockwise adaptive behavior, nonces, . . .)

Full-Keyed Duplex of [DMV17] (1)



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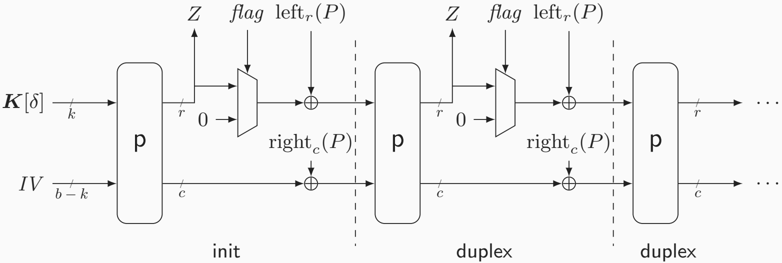
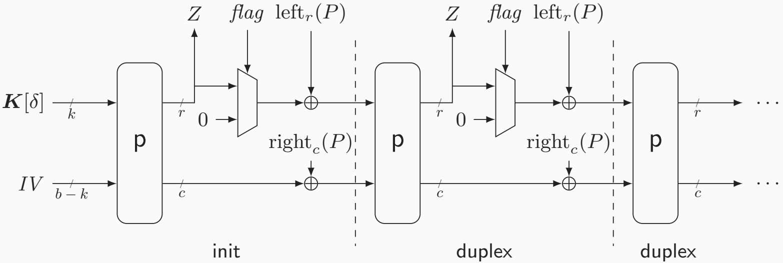


Features

- Multi-user by design: index δ specifies key in array
- Initial state: concatenation of $\boldsymbol{K}[\delta]$ and IV
- Full-state absorption, no padding
- Rephasing: p, Z, P instead of P, p, Z
- Refined adversarial strength

Full-Keyed Duplex of [DMV17] (2)

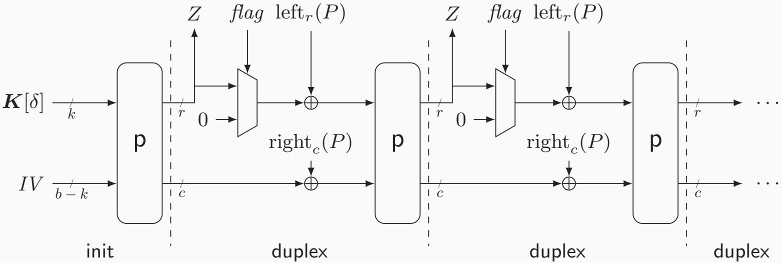


- M: data complexity (calls to construction)
- N: time complexity (calls to primitive)
- Q: number of init calls
- Q_{IV}: max # init calls for single IV
- L: # queries with repeated path (e.g., nonce-violation)
- Ω: # queries with overwriting outer part (e.g., RUP)
- $\nu_{r,c}^{M}$: some multicollision coefficient (often small)

Simplified Security Bound

$$\frac{Q_{IV}N}{2^k} + \frac{(L + \Omega + \nu_{r,c}^{M})N}{2^c}$$

Full-Keyed Duplex of [DM19] (1)



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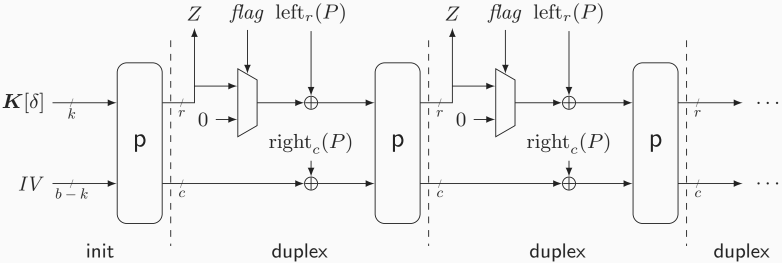
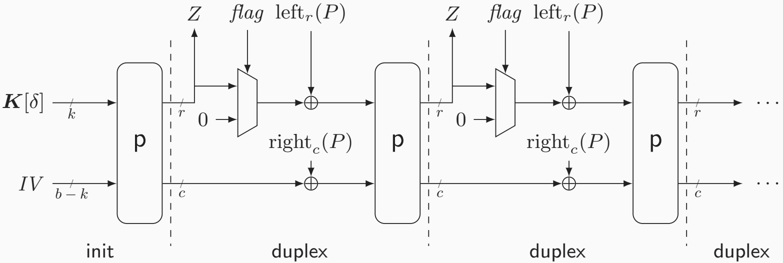


Features

- Initialization can be rotated (not depicted)
- Another rephasing: $Z, P,$ p instead of p, Z, P instead of $P,$ p, Z
- Security analysis in leaky setting
- Even further refined adversarial strength
- Comparable bound

Full-Keyed Duplex of [DM19] (2)

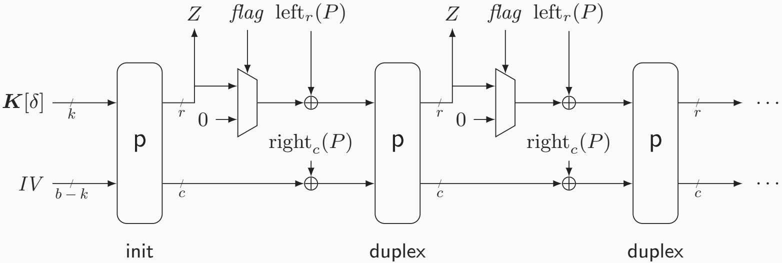


- M: data complexity (calls to construction)
- N: time complexity (calls to primitive)
- Q: number of init calls
- Q_{IV}: max # init calls for single IV
- Q_δ: maximum # init calls for single δ
- L: # queries with repeated path (e.g., nonce-violation)
- Ω: # queries with overwriting outer part (e.g., RUP)
- R: max # duplexing calls for single non-empty path
- $\nu_{r,c}^M$: some multicollision coefficient (often small)

Simplified Security Bound

$$\frac{Q_{IV}N}{2^{k-Q_\delta\lambda}} + \frac{(L+\Omega+\nu_{r,c}^M)N}{2^{c-(R+1)\lambda}}$$

State of Affairs



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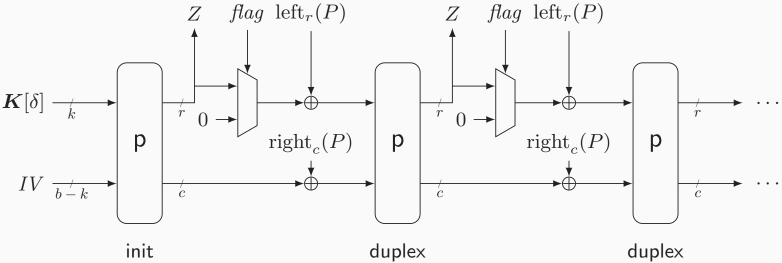

Scheme: versatile but complex

- What about these rephasings?
- What about the flag?

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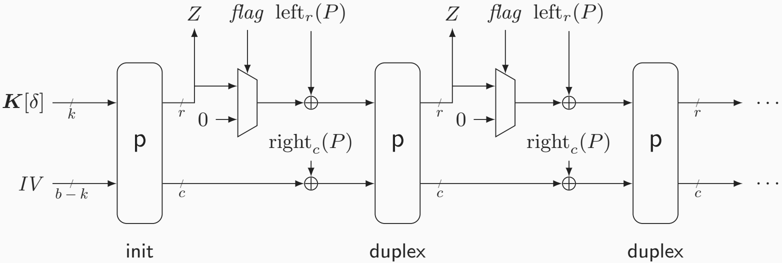

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Security bounds: strong but complex

- Security parameters hard to understand
- Bound quickly misunderstood
- Unclear how use case affects bound

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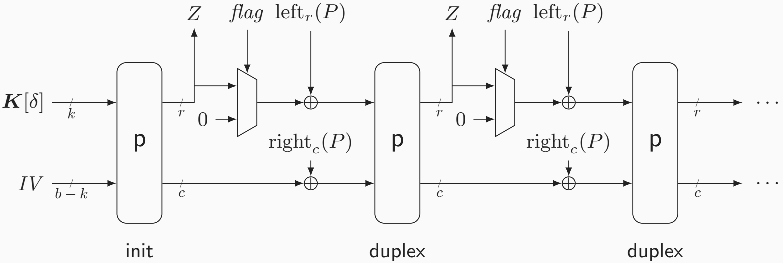

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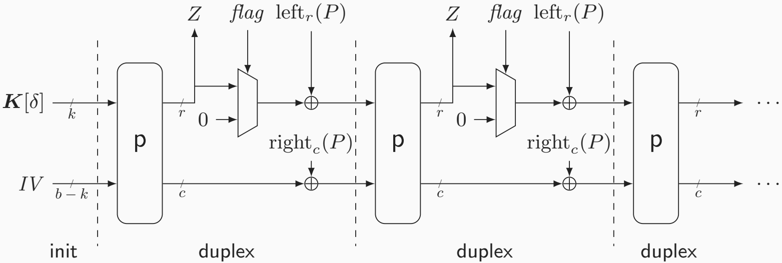
This work: explanation of the duplex, its security, and some applications

Understanding Duplex Design

Generalized Keyed Duplex



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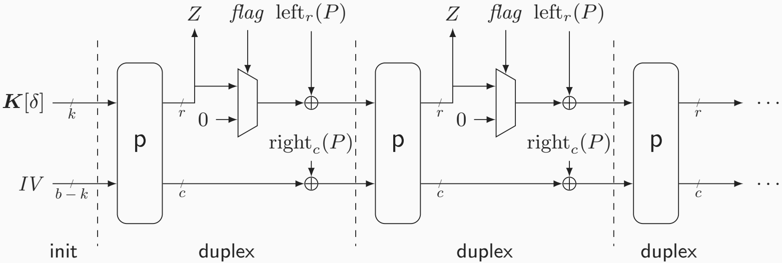

Features

- Basically the scheme of [DMV17] and [DM19], but:
  - including possible initial state rotation (not depicted)
  - yet another rephasing

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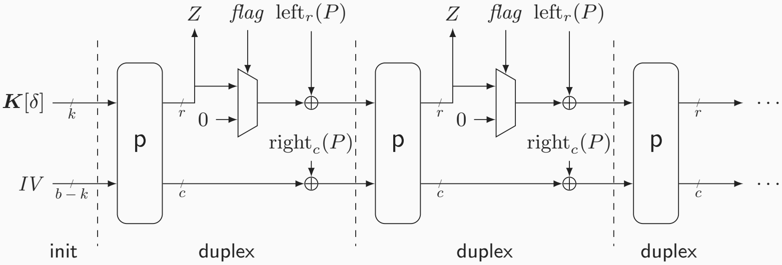
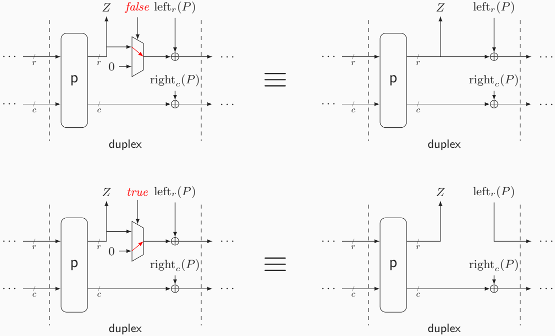


Features

- Basically the scheme of [DMV17] and [DM19], but:
  - including possible initial state rotation (not depicted)
  - yet another rephasing
- Security results of [DMV17] and [DM19] carry over

Generalized Keyed Duplex: Understanding Flagging (1)



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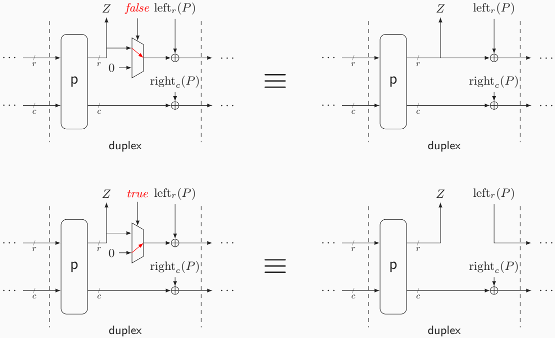

- Typical use case: authenticated encryption using duplex

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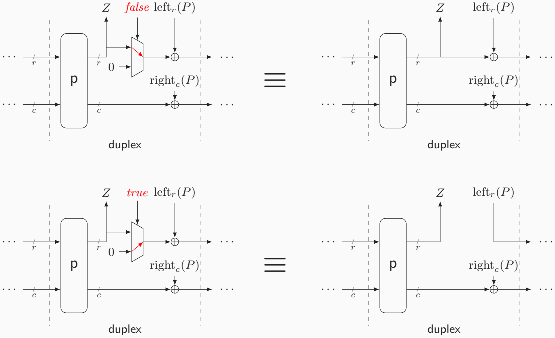

- Typical use case: authenticated encryption using duplex
- Security decreases for increasing number of calls with $\mathit{flag} = \mathit{true}$

Generalized Keyed Duplex: Understanding Flagging (2)

- Consider extreme simplification of SpongeWrap authenticated encryption
- Key K, plaintext P, ciphertext C, and tag T all r bits; nonce U c bits
- General case will be discussed later in this presentation

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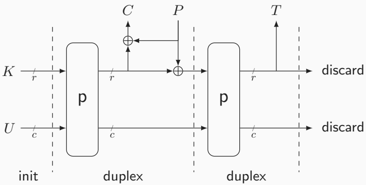
Encryption



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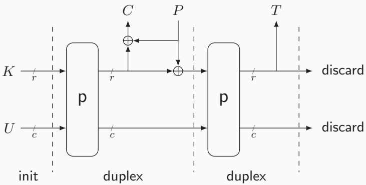

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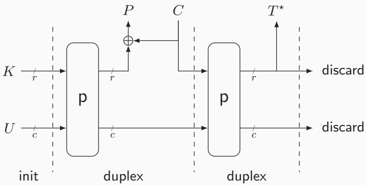

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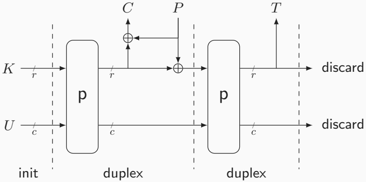

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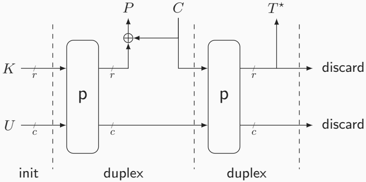

- Duplex call with $\mathit{flag} = \mathit{true}$ upon decryption
- Adversary can choose C and thus fix outer part to value of its choice

Understanding Duplex Security

Security Model ([DMV17, DM19], with updated initial rotation and rephasing)

```
Algorithm Keyed duplex construction KD[p]$_{\boldsymbol{K}}$
Interface: KD.init
Input: $(\delta, IV) \in \{1, \ldots, \mu\} \times \mathcal{IV}$
Output: $\varnothing$
    $S \leftarrow \mathrm{rot}_\alpha(\boldsymbol{K}[\delta] \parallel IV)$
    return $\varnothing$

Interface: KD.duplex
Input: $(\mathit{flag}, P) \in \{\mathit{true}, \mathit{false}\} \times \{0,1\}^b$
Output: $Z \in \{0,1\}^r$
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$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{KD}[\mathsf{p}]_{\boldsymbol{K}}, \mathsf{p}^{\pm}\ ;\ \mathsf{IXIF}[\mathsf{ro}], \mathsf{p}^{\pm}\right)$$

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- IXIF[ro] is basically random oracle in disguise

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- IXIF[ro] is basically random oracle in disguise
- If KD[p]$_{\boldsymbol{K}}$ is hard to distinguish from IXIF[ro] for certain bound on adversarial resources, KD[p]$_{\boldsymbol{K}}$ roughly “behaves like” random oracle

Security Model ([DMV17, DM19], with updated initial rotation and rephasing)

```
Algorithm Keyed duplex construction KD[p]_K
Interface: KD.init
Input: $(\delta, IV) \in \{1, \ldots, \mu\} \times \mathcal{IV}$
Output: $\varnothing$
    $S \leftarrow \mathrm{rot}_\alpha(\boldsymbol{K}[\delta] \parallel IV)$
    return $\varnothing$

Interface: KD.duplex
Input: $(\mathit{flag}, P) \in \{\mathit{true}, \mathit{false}\} \times \{0,1\}^b$
Output: $Z \in \{0,1\}^r$
    $S \leftarrow \mathsf{p}(S)$
    $Z \leftarrow \mathrm{left}_r(S)$
    $S \leftarrow S \oplus [\mathit{flag}] \cdot (Z \| 0^{b-r}) \oplus P$
    return $Z$
```

```
Algorithm Ideal extendable input function IXIF[ro]
Interface: IXIF.init
Input: $(\delta, IV) \in \{1, \ldots, \mu\} \times \mathcal{IV}$
Output: $\varnothing$
    $\mathit{path} \leftarrow \mathrm{encode}[\delta] \parallel IV$
    return $\varnothing$

Interface: IXIF.duplex
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- Bound on adversarial resources is in turn determined by use case!

Security Bounds From [DMV17] and [DM19]

- M: data complexity (calls to construction)
- N: time complexity (calls to primitive)
- Q: number of init calls
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Simplified Security Bound

$$\frac{Q_{IV}N}{2^k} + \frac{(L + \Omega + \nu_{r,c}^{M})N}{2^c}$$

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Actual Security Bounds (Retained)

- [DMV17]:

$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}) \leq \frac{(L+\Omega)N}{2^c} + \frac{2\nu_{r,c}^{2(M-L)}(N+1)}{2^c} + \frac{\binom{L+\Omega+1}{2}}{2^c} + \frac{(M-L-Q)Q}{2^b-Q} + \frac{M(M-L-1)}{2^b} + \frac{Q(M-L-Q)}{2^{\min\{c+k,\max\{b-\alpha,c\}\}}} + \frac{Q_{IV}N}{2^k} + \frac{\binom{\mu}{2}}{2^k}$$

- [DM19] (with one simplification):

$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}) \leq \frac{(L+\Omega)N}{2^c} + \frac{2\nu_{r,c}^{M}(N+1)}{2^c} + \frac{\nu_{r,c}^{M}(L+\Omega)+\binom{L+\Omega}{2}}{2^c} + \frac{\binom{M-L-Q}{2}+(M-L-Q)(L+\Omega)}{2^b} + \frac{\binom{M+N}{2}+\binom{N}{2}}{2^b} + \frac{Q(M-Q)}{2^{\min\{c+k,\max\{b-\alpha,c\}\}}} + \frac{Q_{IV}N}{2^k} + \frac{\binom{\mu}{2}}{2^k}$$

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  4. Message authentication: full-state keyed sponge and Ascon-PRF
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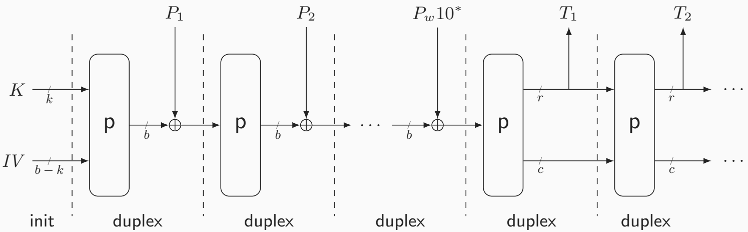
Full-State Keyed Sponge [BDPV12]



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Algorithm Full-state keyed sponge FSKS[p]

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Input: $(K, IV, P) \in \{0,1\}^k \times \mathcal{IV} \times \{0,1\}^*$
Output: $T \in \{0,1\}^t$
Underlying keyed duplex: KD[p]$_{(K)}$
  $(P_1, P_2, \ldots, P_w) \leftarrow \mathrm{pad}_b^{10^*}(P)$
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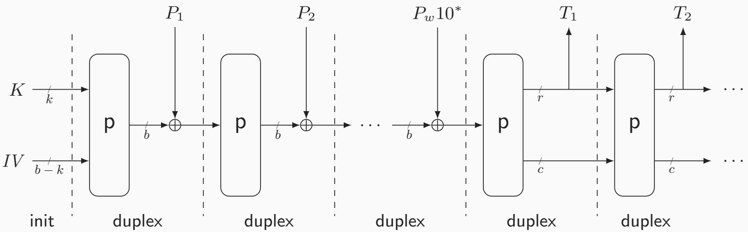

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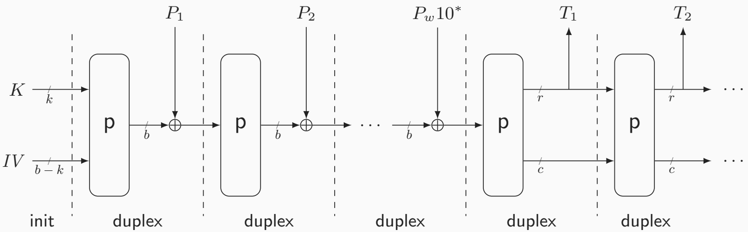

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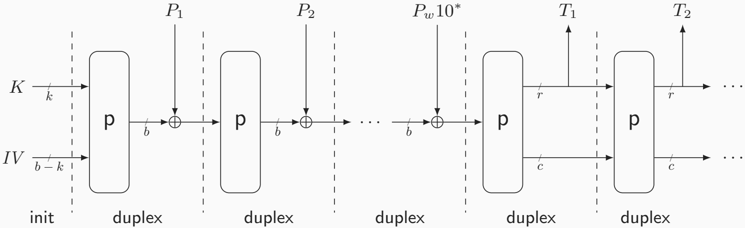

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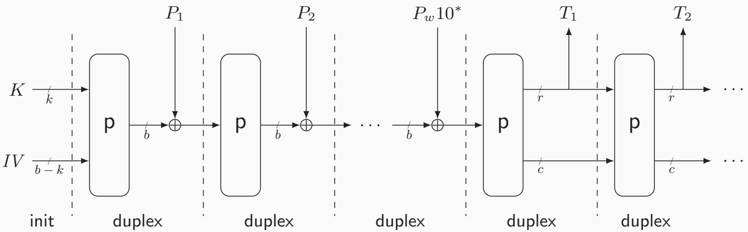

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influence of L

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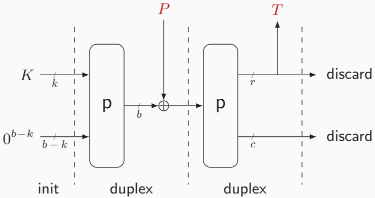

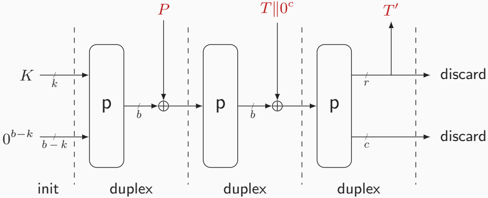

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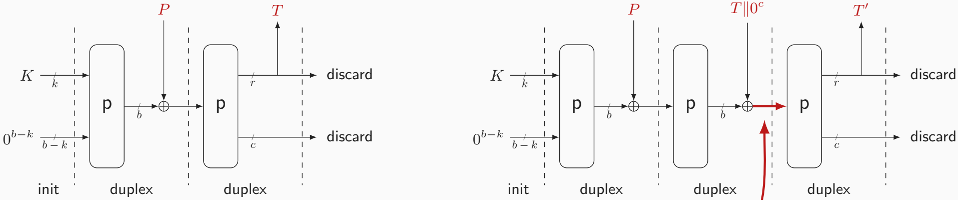

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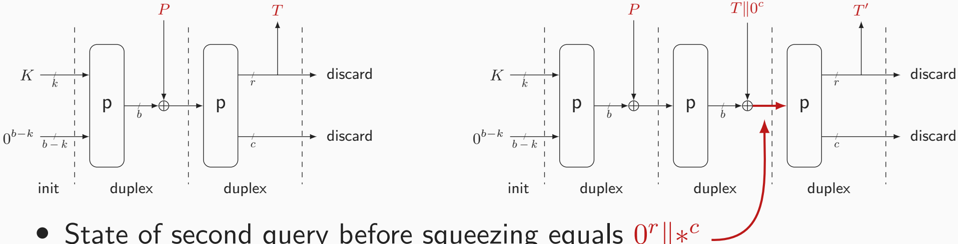

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- Key recovery attack:
  - Make q twin queries as above and N primitive queries of form $0^r \| *^c$
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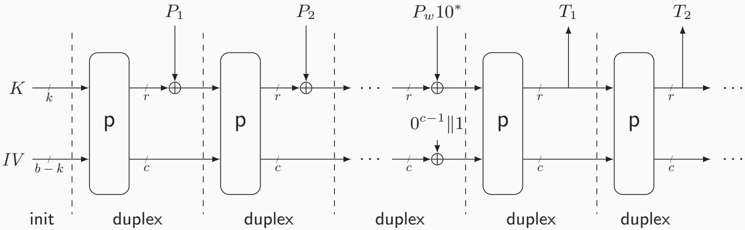
Ascon-PRF [DEMS21]



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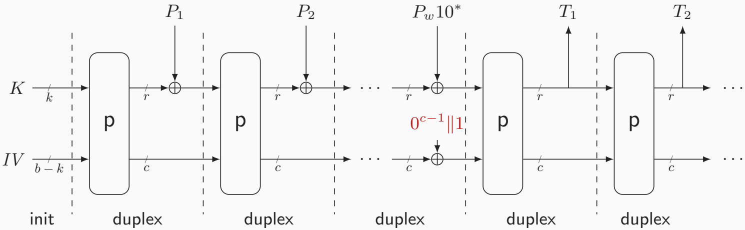

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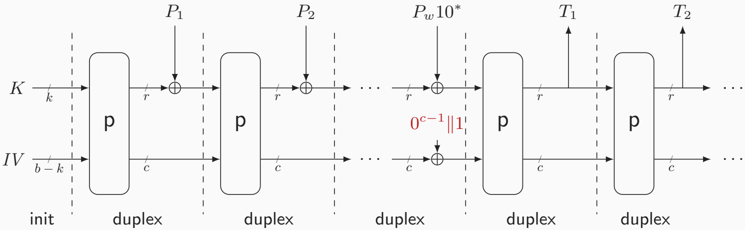

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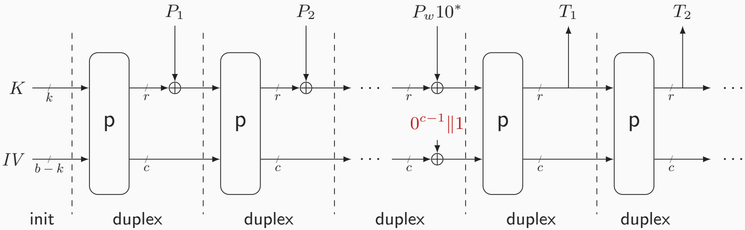
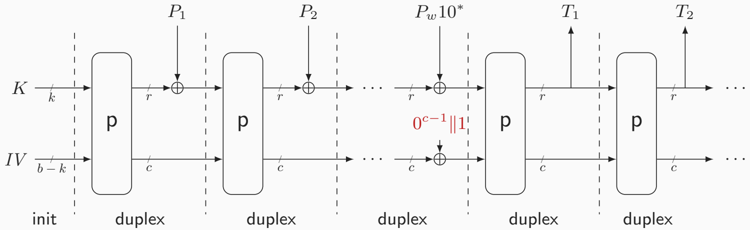
  - Repeated paths may still occur. . .

Ascon-PRF [DEMS21]



- Input: key K, initial value IV, message P
- Output: tag T
- Domain separation solves problem of repeated paths
  - Repeated paths may still occur. . .
  - . . . but adversary cannot exploit them

Ascon-PRF [DEMS21]



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- Output: tag T
- Domain separation solves problem of repeated paths
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  - . . . but adversary cannot exploit them
- Dominant term $\frac{(q-1)N+\binom{q}{2}}{2^c}$ disappears

Conclusion

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Generalized Keyed Duplex

- Versatile construction but application not always clear
- Five representative use cases
- Further use cases: PRNG, PBKDF, . . .
- Generic security of ISAP v2 follows from duplex and SuKS [DEM$^+$20]
- Caution: all presented results only hold in random permutation model

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Much More in Paper

- More detailed explanation on duplex, multicollisions, applications, . . .
- Application of bounds of both [DMV17] and [DM19] to use cases
- Multi-user security

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Thank you for your attention!

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Supporting Slides

Understanding Duplex Design

Generalized Keyed Duplex: Phasing

A	P	S	A	P	S	A	P	S	A	···

Generalized Keyed Duplex: Phasing

	A	P	S	A	P	S	A	P	S	A	· · ·
[BDPV11a]	init			duplex			duplex			· · ·	

- [BDPV11a]: duplex security reduced to sponge indifferentiability

Generalized Keyed Duplex: Phasing

	A	P	S	A	P	S	A	P	S	A	· · ·
[BDPV11a]	init			duplex			duplex			· · ·	
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- [BDPV11a]: duplex security reduced to sponge indifferentiability
- [MRV15]: same structure but tighter bound

Generalized Keyed Duplex: Phasing

<table>
<tr><th></th><th>A</th><th>P</th><th>S</th><th>A</th><th>P</th><th>S</th><th>A</th><th>P</th><th>S</th><th>A</th><th>···</th></tr>
<tr><td>[BDPV11a]</td><td colspan="3">init</td><td colspan="3">duplex</td><td colspan="3">duplex</td><td colspan="2">···</td></tr>
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	A	P	S	A	P	S	A	P	S	A	⋯
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<tr><td>now</td><td>init</td><td colspan="3">duplex</td><td colspan="3">duplex</td><td colspan="3">duplex</td><td>···</td></tr>
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- [MRV15]: same structure but tighter bound
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- [DM19]: security analysis in leaky setting, include upcoming p
- now: seemingly most useful phasing

Intermezzo: Multicollision Coefficient

Intermezzo: Multicollision Coefficient $\nu_{r,c}^{M}$ (1)

Definition

- M balls, 2^r bins
- $\nu_{r,c}^{M}$ is smallest x such that $\Pr\left(|\text{fullest bin}| > x\right) \leq \frac{x}{2^c}$

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Intuition Behind Definition

- We often need upper bound on the maximum multicollision in outer part
- Denote this maximum by ν

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- However, if we take $\nu = \nu_{r,c}^{M}$, this happens with probability at most $\frac{\nu}{2^c}$

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- However, if we take $\nu = \nu_{r,c}^{M}$, this happens with probability at most $\frac{\nu}{2^c}$
- This term is negligible compared to the main probability bound

Intermezzo: Multicollision Coefficient $\nu_{r,c}^{M}$ (2)

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Intuition of Behavior

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- If $M \ll 2^r$, all bins will likely be "reasonably" empty

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- If $M \gg 2^r$, there will likely be a bin with around $linear(b) \cdot \frac{M}{2^r}$ balls

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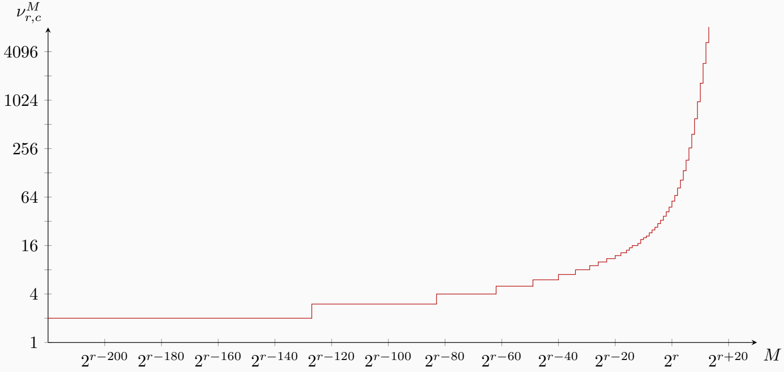
- If $M \ll 2^r$, all bins will likely be "reasonably" empty
- If $M \gg 2^r$, there will likely be a bin with around $linear(b) \cdot \frac{M}{2^r}$ balls
- Formula for $\nu_{r,c}^{M}$, and upper bounds in above 2 cases, derived in [DMV17]
- $\nu_{r,c}^{M}$ is (at most) smallest x that satisfies

$$\frac{2^b e^{-M/2^r} (M/2^r)^x}{(x - M/2^r)x!} \leq 1$$

Intermezzo: Multicollision Coefficient $\nu_{r,c}^{M}$ (3)

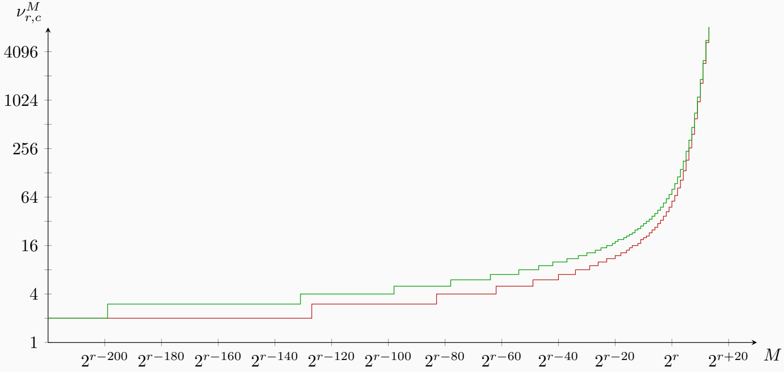
Stairway to Heaven for $b = 256$



$M/2^r$	$\nu_{r,c}^{M}$
2^{-256}	—
2^{-128}	2
2^{-64}	4
2^{-32}	8
2^{-16}	14
2^{-8}	23
2^{0}	57
2^{8}	601
2^{16}	70205
2^{19}	537313

Intermezzo: Multicollision Coefficient $\nu_{r,c}^M$ (3)

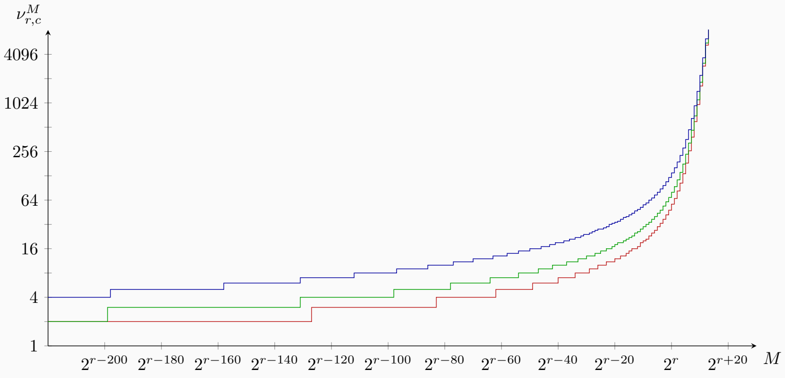
Stairway to Heaven for $b = 256$, $b = 400$



$M/2^r$	$\nu_{r,c}^M$	$\nu_{r,c}^M$
2^{-256}	—	2
2^{-128}	2	4
2^{-64}	4	7
2^{-32}	8	12
2^{-16}	14	21
2^{-8}	23	34
2^{0}	57	80
2^{8}	601	707
2^{16}	70205	71484
2^{19}	537313	540887

Intermezzo: Multicollision Coefficient $\nu_{r,c}^{M}$ (3)

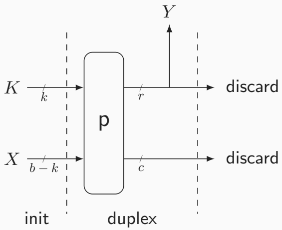
Stairway to Heaven for $b = 256$, $b = 400$, $b = 800$



$M/2^r$	$\nu_{r,c}^{M}$	$\nu_{r,c}^{M}$	$\nu_{r,c}^{M}$
2^{-256}	—	2	4
2^{-128}	2	4	7
2^{-64}	4	7	12
2^{-32}	8	12	23
2^{-16}	14	21	40
2^{-8}	23	34	64
2^{0}	57	80	139
2^{8}	601	707	944
2^{16}	70205	71484	74119
2^{19}	537313	540887	548194

Use Case 1: Truncated Permutation

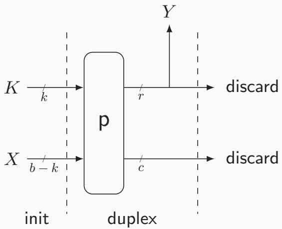
Truncated Permutation



```
**Algorithm** Truncated permutation TP[p]
Input: $(K, X) \in \{0,1\}^k \times \{0,1\}^{b-k}$
Output: $Y \in \{0,1\}^r$
Underlying keyed duplex: KD[p]$_{(K)}$
  KD.init$(1, X)$
  $Y \leftarrow$ KD.duplex$(\mathit{false}, 0^b)$
  return $Y$
```

- PRP-to-PRF conversion: SoP/EDM/EDMD/truncation/STH/...

Truncated Permutation



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  - Sum of externally keyed permutations [CLM19]
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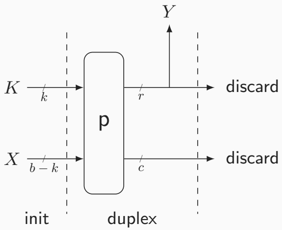

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- Trend towards RP-to-PRF conversion:
  - Sum of externally keyed permutations [CLM19]
  - Permutation-based EDM [DNT21]
- Truncation of externally keyed permutation can be described using duplex

Truncated Permutation: Security (1)

- Consider distinguisher D against PRF security of TP[p]

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{TP}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{TP}[\mathsf{p}]_K, \mathsf{p}^{\pm} \ ; \ \mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right)$$

- D can make q construction queries $+$ N primitive queries

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$= 0$

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$$\hookrightarrow \leq \Delta_{\mathsf{D}'}\left(\mathsf{KD}[\mathsf{p}]_K, \mathsf{p}^{\pm}\ ;\ \mathsf{IXIF}[\mathsf{ro}], \mathsf{p}^{\pm}\right) \qquad \hookrightarrow = 0$$

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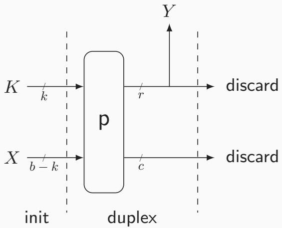
- Triangle inequality:

$$\begin{aligned}
\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{TP}}(\mathsf{D}) &= \Delta_{\mathsf{D}}\left(\mathsf{TP}[\mathsf{p}]_K, \mathsf{p}^{\pm} \;;\; \mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right) \\
&= \Delta_{\mathsf{D}}\left(\mathsf{TP}[\mathsf{KD}[\mathsf{p}]_K], \mathsf{p}^{\pm} \;;\; \mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right) \\
&\le \Delta_{\mathsf{D}}\left(\mathsf{TP}[\mathsf{KD}[\mathsf{p}]_K], \mathsf{p}^{\pm} \;;\; \mathsf{TP}[\mathsf{IXIF}[\mathsf{ro}]], \mathsf{p}^{\pm}\right) + \Delta_{\mathsf{D}}\left(\mathsf{TP}[\mathsf{IXIF}[\mathsf{ro}]], \mathsf{p}^{\pm} \;;\; \mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right)
\end{aligned}$$

$$\hookrightarrow \le \Delta_{\mathsf{D}'}\left(\mathsf{KD}[\mathsf{p}]_K, \mathsf{p}^{\pm} \;;\; \mathsf{IXIF}[\mathsf{ro}], \mathsf{p}^{\pm}\right) \qquad \hookrightarrow = 0$$

- What are the resources of D′?

Truncated Permutation: Security (2)



```
Algorithm Truncated permutation TP[p]
Input: $(K, X) \in \{0,1\}^k \times \{0,1\}^{b-k}$
Output: $Y \in \{0,1\}^r$
Underlying keyed duplex: $\mathsf{KD}[\mathsf{p}]_{(K)}$
    KD.init$(1, X)$
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```

resources of D′	in terms of	resources of D
M: data complexity (calls to construction)		
N: time complexity (calls to primitive)		
Q: number of init calls		
Q_{IV}: max # init calls for single IV		
L: # queries with repeated path		
Ω: # queries with overwriting outer part		

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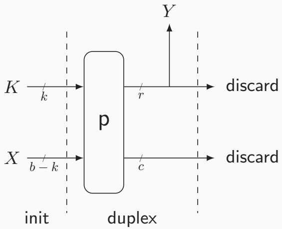

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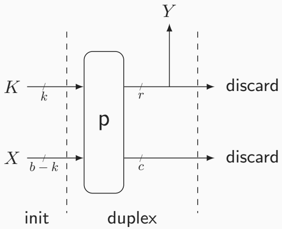

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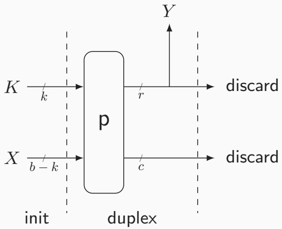

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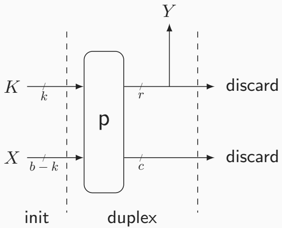
L: # queries with repeated path		
Ω: # queries with overwriting outer part		

Truncated Permutation: Security (2)



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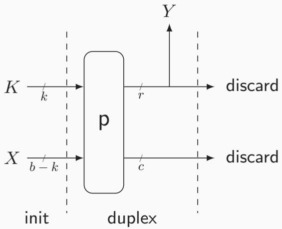

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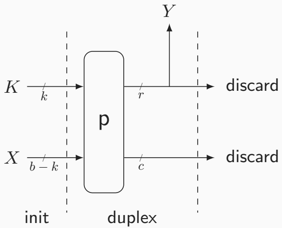

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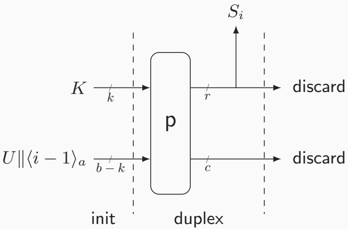
From [DMV17] (in single-user setting): $\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}') \leq \frac{2\nu_{r,c}^{2q}(N+1)}{2^c} + \frac{2\binom{q}{2}}{2^b} + \frac{N}{2^k}$

Use Case 2: Parallel Keystream Generation

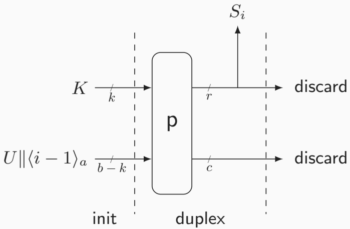
Parallel Keystream Generation



```
Algorithm Parallel keystream generation P-SC[p]
Input: $(K, U, \ell) \in \{0,1\}^k \times \{0,1\}^{b-k-a} \times \{0, \ldots, r2^a\}$
Output: $S \in \{0,1\}^\ell$
Underlying keyed duplex: $\mathsf{KD}[\mathsf{p}]_{(K)}$
  $S \leftarrow \varnothing$
  for $i = 1, \ldots, \lceil \ell / r \rceil$ do
      KD.init($1, U \| \langle i-1 \rangle_a$)
      $S \leftarrow S \,\|\,$ KD.duplex($\mathit{false}, 0^b$)
  return $\mathrm{left}_\ell(S)$
```

- Input: key K, nonce U
- Output: keystream S of requested length

Parallel Keystream Generation



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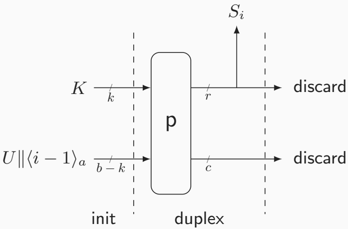
- Input: key K, nonce U
- Output: keystream S of requested length
- P-SC[p] can be seen as TP[p] in counter mode

Parallel Keystream Generation



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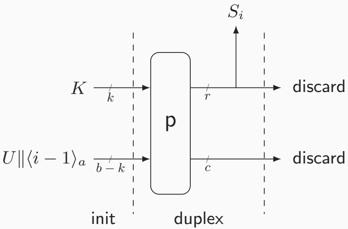
- Output: keystream S of requested length
- P-SC[p] can be seen as TP[p] in counter mode
- PRF security of P-SC[p] easily follows:

Parallel Keystream Generation



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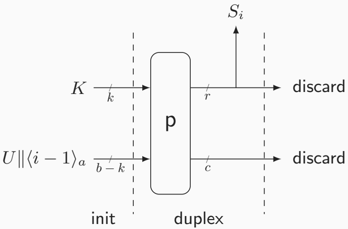
Parallel Keystream Generation



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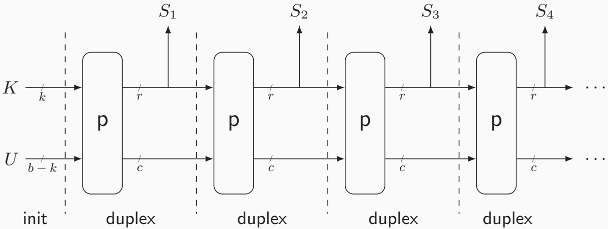
```

- Input: key K, nonce U
- Output: keystream S of requested length
- P-SC[p] can be seen as TP[p] in counter mode
- PRF security of P-SC[p] easily follows:
  - TP[p] behaves like a PRF (up to good bound)
  - Counter mode with a PRF generates uniform random keystream (provided nonce/counter never repeats)

Use Case 3: Sequential Keystream Generation

Sequential Keystream Generation



- Input: key K, nonce U
- Output: keystream S of requested length

Algorithm Sequential keystream generation S-SC[p]

```
Input: $(K, U, \ell) \in \{0,1\}^k \times \{0,1\}^{b-k} \times \mathbb{N}$
Output: $S \in \{0,1\}^\ell$
Underlying keyed duplex: KD[p]$_{(K)}$
  $S \leftarrow \varnothing$
  KD.init$(1, U)$
  for $i = 1, \ldots, \lceil \ell / r \rceil$ do
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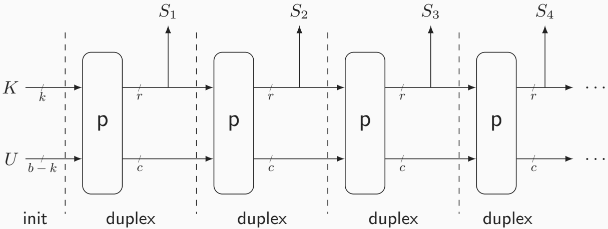
  return $\mathrm{left}_\ell(S)$
```

Sequential Keystream Generation



- Input: key K, nonce U
- Output: keystream S of requested length
- PRF security of S-SC[p]:
  - Comparable analysis as for TP[p]

Algorithm Sequential keystream generation S-SC[p]

```
Input: $(K, U, \ell) \in \{0,1\}^k \times \{0,1\}^{b-k} \times \mathbb{N}$
Output: $S \in \{0,1\}^\ell$
Underlying keyed duplex: $\mathsf{KD}[\mathsf{p}]_{(K)}$
  $S \leftarrow \varnothing$
  $\mathsf{KD.init}(1, U)$
  for $i = 1, \ldots, \lceil \ell / r \rceil$ do
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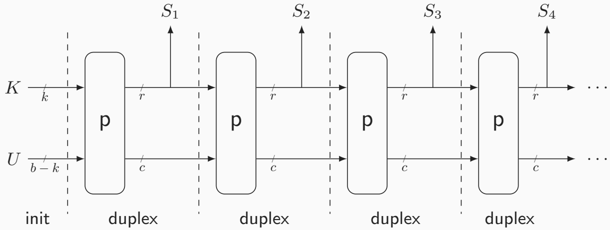
```

Sequential Keystream Generation



- Input: key K, nonce U
- Output: keystream S of requested length
- PRF security of S-SC[p]:
  - Comparable analysis as for TP[p]
  - Resources of D′ slightly differ

Algorithm Sequential keystream generation S-SC[p]

```
Input: $(K, U, \ell) \in \{0,1\}^k \times \{0,1\}^{b-k} \times \mathbb{N}$
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```

Sequential Keystream Generation: Security

- Consider distinguisher D against PRF security of S-SC[p]

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{S\text{-}SC}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{S\text{-}SC}[\mathsf{p}]_K, \mathsf{p}^{\pm} \,;\, \mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right)$$

- D can make q construction queries (total σ blocks) + N primitive queries

Sequential Keystream Generation: Security

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- Triangle inequality: $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{S\text{-}SC}}(\mathsf{D}) \leq \Delta_{\mathsf{D}'}\left(\mathsf{KD}[\mathsf{p}]_K, \mathsf{p}^{\pm}\ ;\ \mathsf{IXIF}[\mathsf{ro}], \mathsf{p}^{\pm}\right)$

Sequential Keystream Generation: Security

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Sequential Keystream Generation: Security

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Sequential Keystream Generation: Security

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Sequential Keystream Generation: Security

- Consider distinguisher D against PRF security of S-SC[p]

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resources of D′	in terms of	resources of D
M: data complexity (calls to construction)	$\longrightarrow$	σ
N: time complexity (calls to primitive)	$\longrightarrow$	N
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Sequential Keystream Generation: Security

- Consider distinguisher D against PRF security of S-SC[p]

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{S\text{-}SC}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{S\text{-}SC}[\mathsf{p}]_K, \mathsf{p}^{\pm} \ ; \ \mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right)$$

- D can make q construction queries (total σ blocks) + N primitive queries
- Triangle inequality: $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{S\text{-}SC}}(\mathsf{D}) \leq \Delta_{\mathsf{D}'}\left(\mathsf{KD}[\mathsf{p}]_K, \mathsf{p}^{\pm} \ ; \ \mathsf{IXIF}[\mathsf{ro}], \mathsf{p}^{\pm}\right)$
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resources of D′	in terms of	resources of D
M: data complexity (calls to construction)	$\longrightarrow$	σ
N: time complexity (calls to primitive)	$\longrightarrow$	N
Q: number of init calls	$\longrightarrow$	q
Q_{IV}: max # init calls for single IV	$\longrightarrow$	1
L: # queries with repeated path	$\longrightarrow$	0
Ω: # queries with overwriting outer part	$\longrightarrow$	0

Sequential Keystream Generation: Security

- Consider distinguisher D against PRF security of S-SC[p]

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{S\text{-}SC}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{S\text{-}SC}[\mathsf{p}]_K, \mathsf{p}^{\pm} \; ; \; \mathsf{R}^{\mathrm{prf}}, \mathsf{p}^{\pm}\right)$$

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From [DMV17] (in single-user setting):

$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}') \leq \frac{2\nu_{r,c}^{2\sigma}(N+1)}{2^c} + \frac{(\sigma-q)q}{2^b-q} + \frac{2\binom{\sigma}{2}}{2^b} + \frac{q(\sigma-q)}{2^{\min\{c+k,b\}}} + \frac{N}{2^k}$$

Use Case 4: Message Authentication

Ascon-PRF: Security

- Unfortunately, (bounds on) the resources of D' do not change:

Ascon-PRF: Security

- Unfortunately, (bounds on) the resources of D′ do not change:

resources of D′	in terms of	resources of D
M: data complexity (calls to construction)	$\longrightarrow$	σ
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Q: number of init calls	$\longrightarrow$	q
Q_{IV}: max # init calls for single IV	$\longrightarrow$	1
L: # queries with repeated path	$\longrightarrow$	$\leq q-1$
Ω: # queries with overwriting outer part	$\longrightarrow$	0

Ascon-PRF: Security

- Unfortunately, (bounds on) the resources of D′ do not change:

resources of D′	in terms of	resources of D
M: data complexity (calls to construction)	$\longrightarrow$	σ
N: time complexity (calls to primitive)	$\longrightarrow$	N
Q: number of init calls	$\longrightarrow$	q
Q_{IV}: max # init calls for single IV	$\longrightarrow$	1
L: # queries with repeated path	$\longrightarrow$	$\leq q - 1$
Ω: # queries with overwriting outer part	$\longrightarrow$	0

- Improved bound from [DMV17]:
  - Loose bounding in original proof
  - Resolving this loose bounding makes $\frac{(q-1)N+\binom{q}{2}}{2^c}$ vanish

Ascon-PRF: Security

- Unfortunately, (bounds on) the resources of D′ do not change:

resources of D′	in terms of	resources of D
M: data complexity (calls to construction)	$\longrightarrow$	σ
N: time complexity (calls to primitive)	$\longrightarrow$	N
Q: number of init calls	$\longrightarrow$	q
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- Improved bound from [DMV17]:
  - Loose bounding in original proof
  - Resolving this loose bounding makes $\frac{(q-1)N+\binom{q}{2}}{2^c}$ vanish
- Improved bound from [DM19]:
  - Defines additional parameter $\nu_{\text{fix}} \leq L + \Omega$
  - In most cases $\nu_{\text{fix}} = L + \Omega$; for current case $\nu_{\text{fix}} = 0$
  - Dominant term $\frac{(q-1)N+\binom{q}{2}}{2^c}$ never appears in the first place

Ascon-PRF: Implication

Multi-user bound from [DMV17]

$$\mathbf{Adv}^{\mu\text{-prf}}_{\text{Ascon-PRF}}(\mathsf{D}) \leq \frac{2\nu_{r,c}^{2\sigma}(N+1)}{2^c} + \frac{(\sigma-q)q}{2^b-q} + \frac{2\binom{\sigma}{2}}{2^b} + \frac{q(\sigma-q)}{2^{\min\{c+k,b\}}} + \frac{\mu N}{2^k} + \frac{\binom{\mu}{2}}{2^k}$$

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Application to Ascon-PRF Parameters

- $(k, b, c, r) = (128, 320, 192, 128)$
- Assume online complexity of $q, \sigma \ll 2^{64}$ (could be taken higher)
- The multicollision term $\nu^{2^{65}}_{128,192}$ is at most 5

Ascon-PRF: Implication

Multi-user bound from [DMV17]

$$\begin{tikzcd}
\mathbf{Adv}^{\mu\text{-prf}}_{\text{Ascon-PRF}}(\mathsf{D}) \leq \frac{2\nu^{2\sigma}_{r,c}(N+1)}{2^c} \arrow[d, "\leq"] & + & \frac{(\sigma-q)q}{2^b-q} \arrow[d, "\leq"] & + & \frac{2\binom{\sigma}{2}}{2^b} \arrow[d, "\leq"] & + & \frac{q(\sigma-q)}{2^{\min\{c+k,b\}}} \arrow[d, "\leq"] & + & \frac{\mu N}{2^k} \arrow[d, "\leq"] & + & \frac{\binom{\mu}{2}}{2^k} \arrow[d, "\leq"] \\
\frac{10(N+1)}{2^{192}} & + & \frac{2^{128}}{2^{320}} & + & \frac{2^{128}}{2^{320}} & + & \frac{2^{128}}{2^{320}} & + & \frac{\mu N}{2^{128}} & + & \frac{\binom{\mu}{2}}{2^{128}}
\end{tikzcd}$$

Application to Ascon-PRF Parameters

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Multi-user bound from [DMV17]

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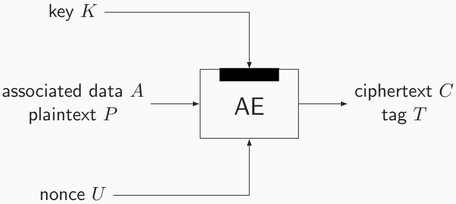
\mathbf{Adv}^{\mu\text{-prf}}_{\textsf{Ascon-PRF}}(\mathsf{D}) \leq \frac{2\nu^{2\sigma}_{r,c}(N+1)}{2^c} \arrow[d, "\leq"] & + & \frac{(\sigma-q)q}{2^b-q} \arrow[d, "\leq"] & + & \frac{2\binom{\sigma}{2}}{2^b} \arrow[d, "\leq"] & + & \frac{q(\sigma-q)}{2^{\min\{c+k,b\}}} \arrow[d, "\leq"] & + & \frac{\mu N}{2^k} \arrow[d, "\leq"] & + & \frac{\binom{\mu}{2}}{2^k} \arrow[d, "\leq"] \\
\frac{10(N+1)}{2^{192}} & + & \frac{2^{128}}{2^{320}} & + & \frac{2^{128}}{2^{320}} & + & \frac{2^{128}}{2^{320}} & + & \frac{\mu N}{2^{128}} & + & \frac{\binom{\mu}{2}}{2^{128}}
\end{tikzcd}$$

Application to Ascon-PRF Parameters

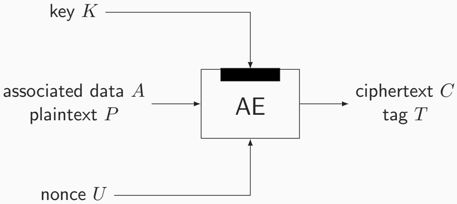
- $(k, b, c, r) = (128, 320, 192, 128)$
- Assume online complexity of $q, \sigma \ll 2^{64}$ (could be taken higher)
- The multicollision term $\nu^{2^{65}}_{128,192}$ is at most 5
- Generic security as long as $N \ll 2^{128}/\mu$

Use Case 5: Authenticated Encryption

Authenticated Encryption



Authenticated Encryption



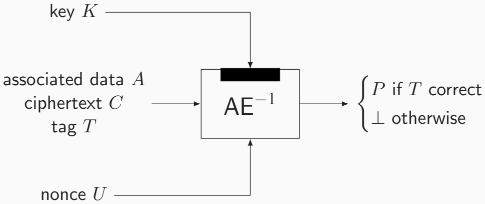
key K

associated data A
plaintext P

AE

ciphertext C
tag T

nonce U



key K

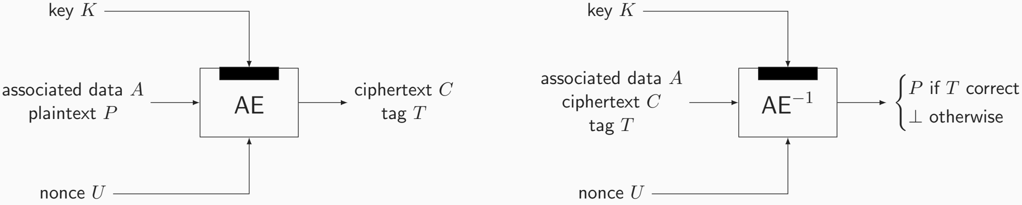
associated data A
ciphertext C
tag T

AE^{-1}

$\begin{cases} P \text{ if } T \text{ correct} \\ \perp \text{ otherwise} \end{cases}$

nonce U

Authenticated Encryption



key K

associated data A
plaintext P

AE

ciphertext C
tag T

nonce U

key K

associated data A
ciphertext C
tag T

AE^{-1}

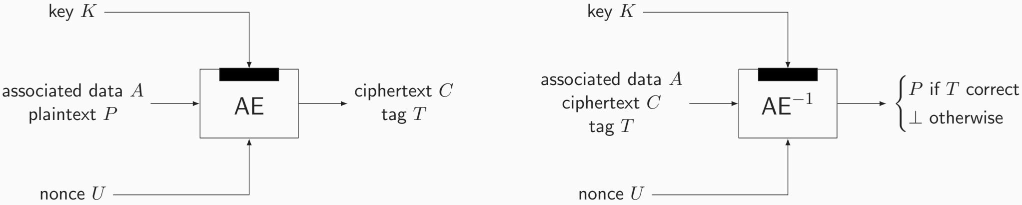
$\begin{cases} P \text{ if } T \text{ correct} \\ \perp \text{ otherwise} \end{cases}$

nonce U

Role of Duplex

- Blockwise construction allows for processing different types of in-/output

Authenticated Encryption



key K

associated data A
plaintext P

AE

nonce U

ciphertext C
tag T

key K

associated data A
ciphertext C
tag T

AE^{-1}

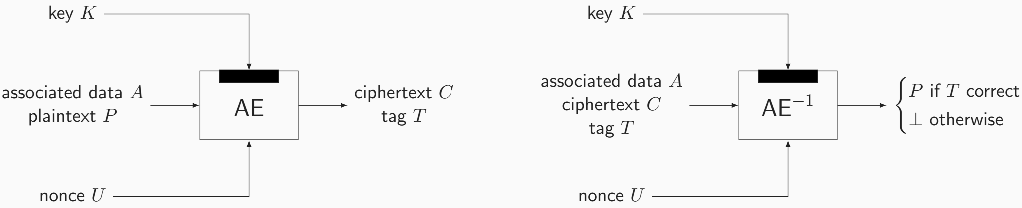
nonce U

$$\begin{cases} P \text{ if } T \text{ correct} \\ \perp \text{ otherwise} \end{cases}$$

Role of Duplex

- Blockwise construction allows for processing different types of in-/output
- Usage of flag makes duplex-style encryption decryptable

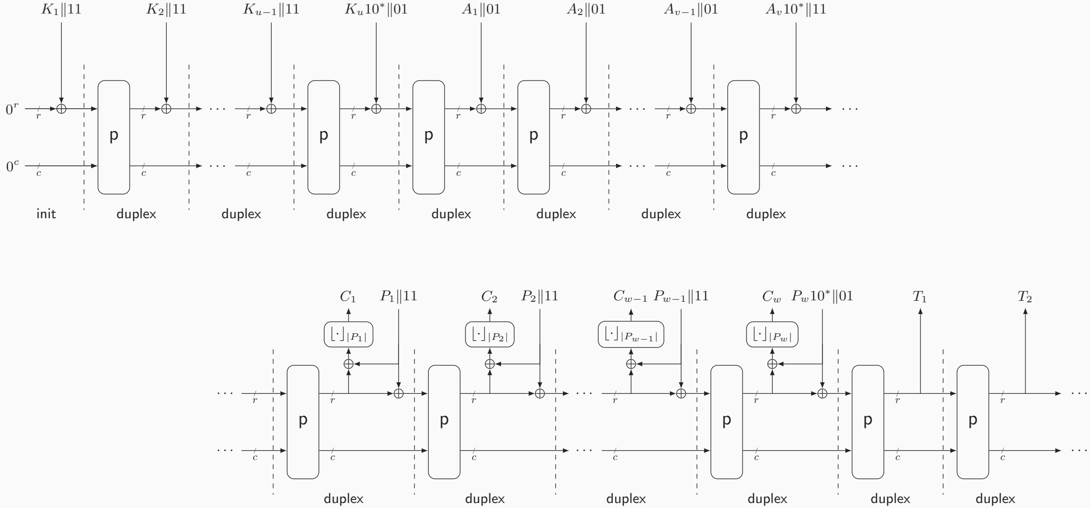
Authenticated Encryption



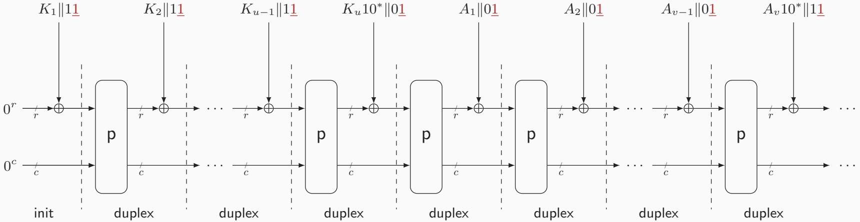
Role of Duplex

- Blockwise construction allows for processing different types of in-/output
- Usage of flag makes duplex-style encryption decryptable
  (Although the flag is not a necessity for this)

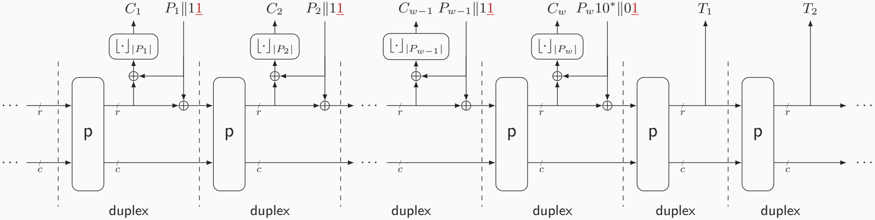
SpongeWrap [BDPV11a]



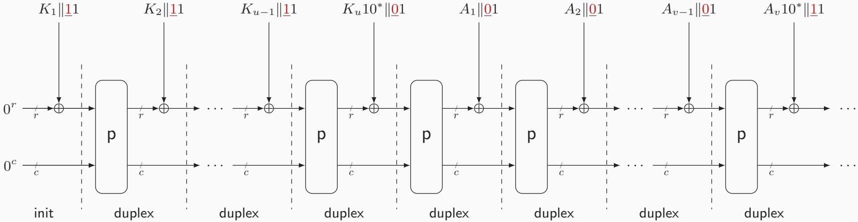
Issues with SpongeWrap



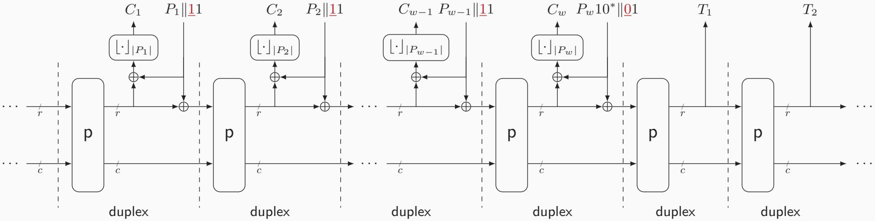
? always 1-padding



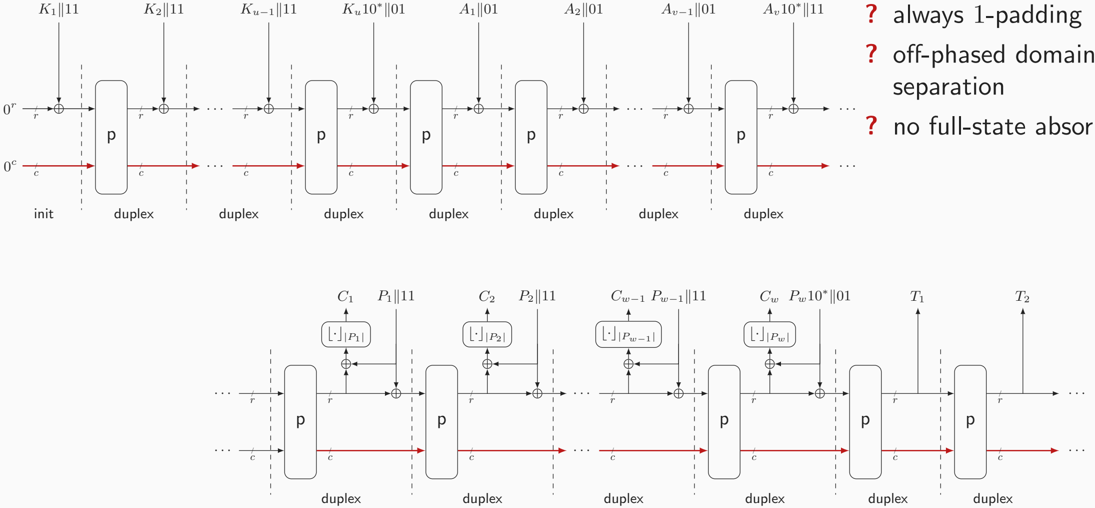
Issues with SpongeWrap



- ? always 1-padding
- ? off-phased domain separation

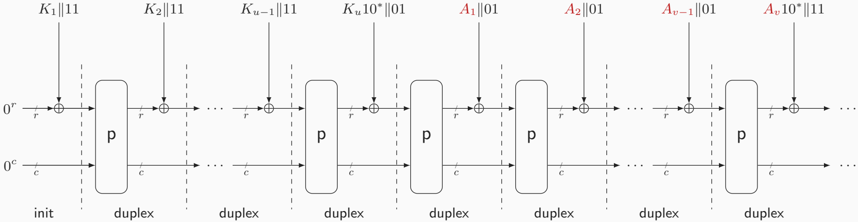


Issues with SpongeWrap

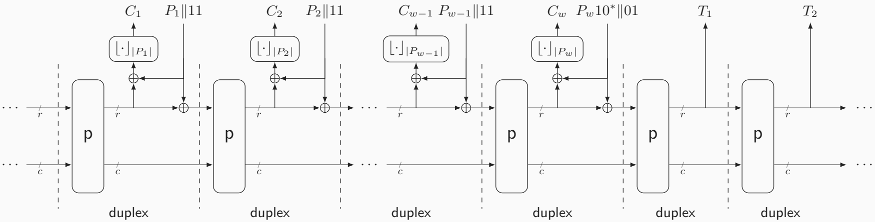


- ? always 1-padding
- ? off-phased domain separation
- ? no full-state absorption

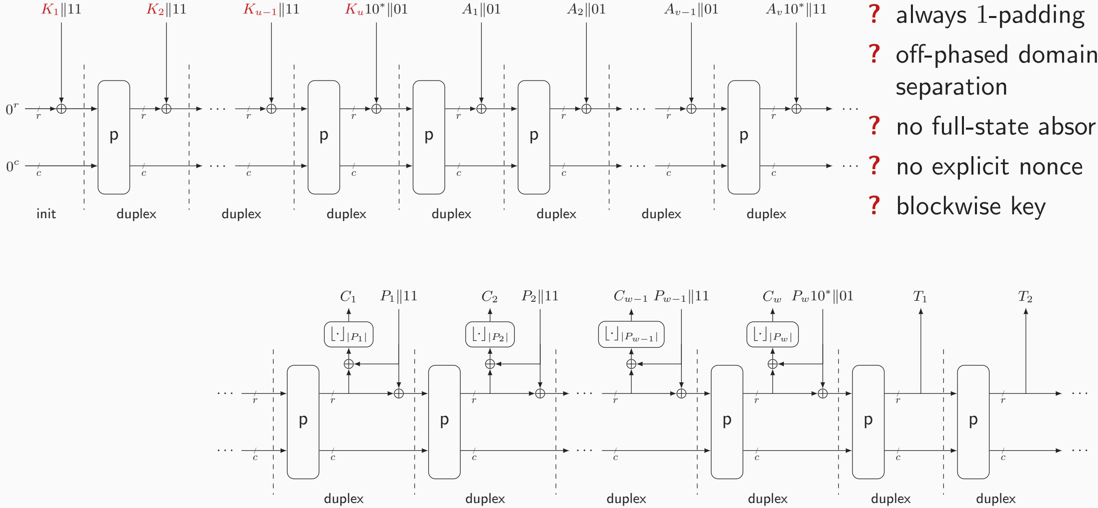
Issues with SpongeWrap



- ? always 1-padding
- ? off-phased domain separation
- ? no full-state absorption
- ? no explicit nonce

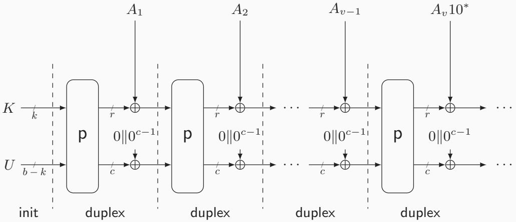


Issues with SpongeWrap

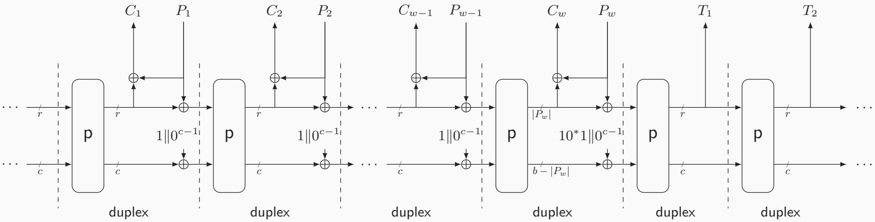


- ? always 1-padding
- ? off-phased domain separation
- ? no full-state absorption
- ? no explicit nonce
- ? blockwise key

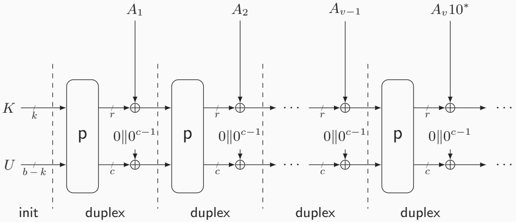
MonkeySpongeWrap: Encryption



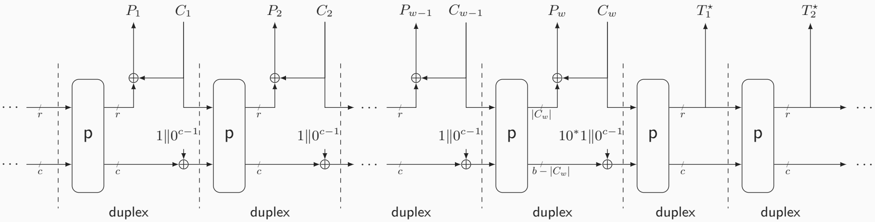
- State initialized using key and nonce
- Cleaned-up and synchronized domain separation
- Spill-over into inner part
- Used in Xoodyak and Gimli (a.o.)



MonkeySpongeWrap: Decryption



- Decryption similar to encryption
- Notable difference:
    - Processing of C
    - Duplexing with $\mathit{flag} = \mathit{true}$



MonkeySpongeWrap: Algorithm

Algorithm MonkeySpongeWrap[p]: ENC

Input: $(K, U, A, P) \in \{0,1\}^k \times \{0,1\}^{b-k} \times \{0,1\}^* \times \{0,1\}^*$
Output: $(C, T) \in \{0,1\}^{|P|} \times \{0,1\}^t$
Underlying keyed duplex: KD[p]$_{(K)}$

```
$(A_1, A_2, \ldots, A_v) \leftarrow \mathrm{pad}_r^{10^*}(A)$
$(P_1, P_2, \ldots, P_w) \leftarrow \mathrm{pad}_r^{10^*}(P)$
$C \leftarrow \varnothing$
$T \leftarrow \varnothing$
KD.init$(1, U)$
for $i = 1, \ldots, v$ do
    KD.duplex$(\mathit{false}, A_i \| 0 \| 0^{c-1})$    ▷ discard output
for $i = 1, \ldots, w$ do
    $C \leftarrow C \ \|$ KD.duplex$(\mathit{false}, P_i \| 1 \| 0^{c-1}) \oplus P_i$
for $i = 1, \ldots, \lceil t/r \rceil$ do
    $T \leftarrow T \ \|$ KD.duplex$(\mathit{false}, 0^b)$
return $(\mathrm{left}_{|P|}(C), \mathrm{left}_t(T))$
```

Algorithm MonkeySpongeWrap[p]: DEC

Input: $(K, U, A, C, T) \in \{0,1\}^k \times \{0,1\}^{b-k} \times \{0,1\}^* \times \{0,1\}^* \times \{0,1\}^t$
Output: $P \in \{0,1\}^{|C|}$ or $\perp$
Underlying keyed duplex: KD[p]$_{(K)}$

```
$(A_1, A_2, \ldots, A_v) \leftarrow \mathrm{pad}_r^{10^*}(A)$
$(C_1, C_2, \ldots, C_w) \leftarrow \mathrm{pad}_r^{10^*}(C)$
$P \leftarrow \varnothing$
$T^\star \leftarrow \varnothing$
KD.init$(1, U)$
for $i = 1, \ldots, v$ do
    KD.duplex$(\mathit{false}, A_i \| 0 \| 0^{c-1})$    ▷ discard output
for $i = 1, \ldots, w$ do
    $P \leftarrow P \ \|$ KD.duplex$(\mathit{true}, C_i \| 1 \| 0^{c-1}) \oplus C_i$
for $i = 1, \ldots, \lceil t/r \rceil$ do
    $T^\star \leftarrow T^\star \ \|$ KD.duplex$(\mathit{false}, 0^b)$
return $\mathrm{left}_t(T) = \mathrm{left}_t(T^\star) \ ? \ \mathrm{left}_{|C|}(P) : \perp$
```

MonkeySpongeWrap: Security (1)

- Consider distinguisher D against AE security of MonkeySpongeWrap[p]

$$\mathbf{Adv}^{\mathrm{ae}}_{\mathsf{MonkeySpongeWrap}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{ENC}[\mathsf{p}]_K, \mathsf{DEC}[\mathsf{p}]_K, \mathsf{p}^{\pm} \;;\; \mathsf{R}^{\mathrm{ae}}, \perp, \mathsf{p}^{\pm}\right)$$

- D can make: q_e encryption queries (total σ_e blocks),
  q_d decryption queries (total σ_d blocks),
  N primitive queries

MonkeySpongeWrap: Security (1)

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- Triangle inequality derivation slightly more involved than before:

$$\mathbf{Adv}^{\mathrm{ae}}_{\mathsf{MonkeySpongeWrap}}(\mathsf{D}) \leq \Delta_{\mathsf{D}'}\left(\mathsf{KD}[\mathsf{p}]_K, \mathsf{p}^{\pm}\ ;\ \mathsf{IXIF}[\mathsf{ro}], \mathsf{p}^{\pm}\right) + \frac{q_d}{2^t}$$

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$$\mathbf{Adv}^{\mathrm{ae}}_{\mathsf{MonkeySpongeWrap}}(\mathsf{D}) = \Delta_{\mathsf{D}}\left(\mathsf{ENC}[\mathsf{p}]_K, \mathsf{DEC}[\mathsf{p}]_K, \mathsf{p}^{\pm} \;;\; \mathsf{R}^{\mathrm{ae}}, \perp, \mathsf{p}^{\pm}\right)$$

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- What are the resources of D′?

MonkeySpongeWrap: Security (2)

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- Encryption calls: unique nonce, *flag* always *false*
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MonkeySpongeWrap: Security (2)

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resources of D′	in terms of	resources of D
M: data complexity (calls to construction)		
N: time complexity (calls to primitive)	$\longrightarrow$	N
Q: number of init calls		
Q_{IV}: max # init calls for single IV		
L: # queries with repeated path		
Ω: # queries with overwriting outer part		

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resources of D′	in terms of	resources of D
M: data complexity (calls to construction)	$\longrightarrow$	$\sigma_e + \sigma_d$
N: time complexity (calls to primitive)	$\longrightarrow$	N
Q: number of init calls	$\longrightarrow$	$q_e + q_d$
Q_{IV}: max # init calls for single IV		
L: # queries with repeated path		
Ω: # queries with overwriting outer part		

MonkeySpongeWrap: Security (2)

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N: time complexity (calls to primitive)	$\longrightarrow$	N
Q: number of init calls	$\longrightarrow$	$q_e + q_d$
Q_{IV}: max # init calls for single IV	$\longrightarrow$	1
L: # queries with repeated path	$\longrightarrow$	$\leq q_d$
Ω: # queries with overwriting outer part		

MonkeySpongeWrap: Security (2)

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M: data complexity (calls to construction)	$\longrightarrow$	$\sigma_e + \sigma_d$
N: time complexity (calls to primitive)	$\longrightarrow$	N
Q: number of init calls	$\longrightarrow$	$q_e + q_d$
Q_{IV}: max # init calls for single IV	$\longrightarrow$	1
L: # queries with repeated path	$\longrightarrow$	$\leq q_d$
Ω: # queries with overwriting outer part	$\longrightarrow$	$\leq \sigma_d - 2q_d$

MonkeySpongeWrap: Security (2)

- D can make: q_e encryption queries (total σ_e blocks),
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Ω: # queries with overwriting outer part	$\longrightarrow$	$\leq \sigma_d - 2q_d$

From [DMV17] (in single-user setting):

$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}') \leq \frac{2\nu_{r,c}^{2\sigma}(N+1)}{2^c} + \frac{\sigma_d N + \binom{\sigma_d}{2}}{2^c} + \frac{(\sigma-q)q}{2^b-q} + \frac{2\binom{\sigma}{2}}{2^b} + \frac{q(\sigma-q)}{2^{\min\{c+k,b\}}} + \frac{N}{2^k}$$

MonkeySpongeWrap: Security (2)

- D can make: q_e encryption queries (total σ_e blocks),
  q_d decryption queries (total σ_d blocks),
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- Encryption calls: unique nonce, *flag* always *false*
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resources of D′	in terms of	resources of D
M: data complexity (calls to construction)	$\longrightarrow$	$\sigma_e + \sigma_d$
N: time complexity (calls to primitive)	$\longrightarrow$	N
Q: number of init calls	$\longrightarrow$	$q_e + q_d$
Q_{IV}: max # init calls for single IV	$\longrightarrow$	1
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attack of Gilbert et al. [GBKR23] “operates” here

MonkeySpongeWrap: Security (2)

- D can make: q_e encryption queries (total σ_e blocks),
  q_d decryption queries (total σ_d blocks),
  N primitive queries
- Encryption calls: unique nonce, *flag* always *false*
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resources of D′	in terms of	resources of D
M: data complexity (calls to construction)	$\longrightarrow$	$\sigma_e + \sigma_d$
N: time complexity (calls to primitive)	$\longrightarrow$	N
Q: number of init calls	$\longrightarrow$	$q_e + q_d$
Q_{IV}: max # init calls for single IV	$\longrightarrow$	1
L: # queries with repeated path	$\longrightarrow$	$\leq q_d$
Ω: # queries with overwriting outer part	$\longrightarrow$	$\leq \sigma_d - 2q_d$

From [DMV17] (in single-user setting):

$$\mathbf{Adv}_{\mathsf{KD}}(\mathsf{D}') \leq \frac{2\nu_{r,c}^{2\sigma}(N+1)}{2^c} + \frac{\sigma_d N + \binom{\sigma_d}{2}}{2^c} + \frac{(\sigma-q)q}{2^b-q} + \frac{2\binom{\sigma}{2}}{2^b} + \frac{q(\sigma-q)}{2^{\min\{c+k,b\}}} + \frac{N}{2^k}$$

attack of Gilbert et al. [GBKR23] "operates" here, with $\sigma_d, N \approx 2^{3c/4}$